Tracking

AE353

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Bretl

We have learned to apply linear state feedback:

$$u = -Kx.$$

When it works (i.e., when all eigenvalues of $A - BK$ have negative real part), linear state feedback does exactly one thing:

$$x(t) \to 0 \quad \text{as} \quad t \to \infty.$$

Equivalently, it makes

$$m(t) \to m_e \quad \text{as} \quad t \to \infty. \qquad \leftarrow \text{remember: } x = u - u_e$$

Since the equilibrium point m_e has to be chosen in advance and cannot be changed, then our controllers cannot (yet) do what we want them to do – for example, make the wheel reach <u>any</u> target angle or make the cat-catching robot reach <u>any</u> target position. For that, we would want

$$m(t) \to m_{des} \quad \text{as} \quad t \to \infty$$

where m_{des} is a desired state that we can change on the fly. We call this <u>tracking</u> (or "reference tracking").

$\dot{m} = f(m, n)$

HOW CAN YOU TRACK (WITHOUT OBSERVER)

① Linearize about m_e, n_e as usual

$\dot{x} = Ax + Bu \qquad x = m - m_e \qquad u = n - n_e$

m_e and n_e must remain constant

② Compute a desired state

CHOOSE

m_{des} ← m_{des} can change on the fly, but must differ from m_e only in those variables that can be tracked

DEFINE

$x_{des} = m_{des} - m_e$

IMPLEMENT

$u = -K(x - x_{des})$

$$\dot{m} = f(m, n)$$
$$o = g(m, n)$$

HOW CAN YOU TRACK (WITH OBSERVER)

(1) Linearize about m_e, n_e as usual

$$\dot{x} = Ax + Bu \qquad x = m - m_e \qquad u = n - n_e \qquad y = o - g(m_e, n_e)$$
$$y = Cx$$

m_e and n_e must remain constant

(2) Compute a desired state

CHOOSE

$$m_{des}$$

← m_{des} can change on the fly, but must differ from m_e only in those variables that can be tracked

DEFINE

$$x_{des} = m_{des} - m_e$$

IMPLEMENT

$$u = -K(\hat{x} - x_{des})$$

SEE REFERENCE PAGES FOR DETAILS ON HOW TO DECIDE WHAT CAN AND CANNOT BE TRACKED