Observers (Part 2)

AE353

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Bretl

$$\dot{x} = A\overset{\substack{\text{state}\\ \downarrow}}{x} + B\overset{\substack{\text{input}\\ \downarrow}}{u} \qquad \leftarrow \text{dynamic model}$$

$$\underset{\substack{\uparrow \\ \text{output}}}{y} = Cx \qquad \leftarrow \text{sensor model}$$

$$u = -K\hat{x} \qquad \leftarrow \text{controller}$$

$$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y) \qquad \leftarrow \text{observer}$$

HOW TO IMPLEMENT IT ?

$$\dot{x} = Ax + Bu \qquad\qquad u = -K\hat{x}$$
$$y = Cx \qquad\qquad \dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$$

RESET $\left\{ \hat{x}(0) = 0 \right.$

RUN
$$\left\{ \begin{array}{l} \vdots \\ u(t) = -K\hat{x}(t) \\ \hat{x}(t+\Delta t) \approx \hat{x}(t) + \Delta t \left(A\hat{x}(t) + Bu(t) - L\left(C\hat{x}(t) - y(t)\right)\right) \\ \vdots \end{array} \right.$$

WHEN DOES IT WORK?

$$\dot{x} = Ax + Bu \qquad u = -K\hat{x}$$
$$y = Cx \qquad \dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$$

$$x_{err} = \hat{x} - x$$ ← does this converge to zero or not?

$$\dot{x}_{err} = \dot{\hat{x}} - \dot{x}$$
$$= (A\hat{x} + Bu - L(C\hat{x} - y)) - (Ax + Bu)$$
$$= A\hat{x} + \cancel{Bu} - LC\hat{x} + Ly - Ax - \cancel{Bu}$$
$$= A\hat{x} - Ax - LC\hat{x} + LCx$$
$$= A(\hat{x} - x) - LC(\hat{x} - x)$$
$$= (A - LC)(\hat{x} - x)$$

$$\boxed{\dot{x}_{err} = (A - LC)\, x_{err}}$$

HOW TO CHOOSE L ?

FACTS

$$(M+N)^T = M^T + N^T$$
$$(MN)^T = N^T M^T$$
$$\det(M) = \det(M^T)$$

$$\dot{x} = (A - BK)x$$

$$0 = \det(sI - (A - BK))$$

K=place_poles(A,B,Pc).gain_matrix

$$\dot{x}_{err} = (A - LC)x_{err}$$

$$0 = \det(sI - (A - LC))$$
$$= \det\left((sI - (A-LC))^T \right)$$
$$= \det\left((sI)^T - (A - LC)^T \right)$$
$$= \det\left(sI - \left((A)^T - (LC)^T \right) \right)$$
$$= \det\left(sI - (A^T - C^T L^T) \right)$$

L = place_poles(A.T, C.T, Po).gain_matrix.T

WHEN IS OBSERVER DESIGN POSSIBLE?

$\dot{x} = (A - BK)x$

controllable when

$W_c = [B \quad AB \quad A^2B \quad \cdots \quad A^{n-1}B]$

is full rank

$\dot{x}_{err} = (A - LC)x_{err}$

observable when

$$W_o = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} \longleftrightarrow [C^T \quad A^TC^T \quad (A^T)^2C^T \quad \cdots \quad]$$

is full rank