Observers (Part 1)

AE353

Spring 2025

Bretl

$$\dot{x} = Ax + Bu$$

until now:

$$u = -Kx$$

assumes we know x – often we don't

from now on:

$$u = -K\hat{x}$$

our <u>estimate</u> of x – how do we get this?

SEE EXAMPLE (20250312/WheelDemo-WithSensors)

$$\dot{x} = A\overset{\text{state}}{x} + B\overset{\text{input}}{u}$$

$$\underset{\text{output}}{y} = Cx$$

$$x = \begin{bmatrix} q - q_e \\ v - v_e \end{bmatrix} \qquad u = \begin{bmatrix} \tau - \tau_e \end{bmatrix}$$

$$y = \begin{bmatrix} q - q_e \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

HOW TO GO FROM y TO x?

$$\cancel{y = [q]}$$

$$Cx = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} q - q_e \\ v - v_e \end{bmatrix} = \begin{bmatrix} q - q_e \end{bmatrix}$$

SEE EXAMPLE (20250312/WheelDemo-WithSensors)

state input

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

output

$$x = \begin{bmatrix} q - q_e \\ v - v_e \end{bmatrix} \qquad u = \begin{bmatrix} \tau - \tau_e \end{bmatrix}$$

$$y = \begin{bmatrix} q - q_e \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

HOW TO GO FROM y TO x?

$$v(t) \approx \frac{q(t) - q(t-\Delta t)}{\Delta t} = \frac{(q(t) - q_e) - (q(t-\Delta t) - q_e)}{\Delta t}$$

$$\cancel{\hat{x} = C^{-1} y}$$

$$\hat{x} = C^{\dagger} y = (C^T C)^{-1} C^T y$$

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

how to go from y to x?

Take inspiration from state feedback:

$$\dot{x} = Ax$$ ← what x does without control

$$\dot{x} = \underbrace{Ax - BK(x - \overbrace{x_{desired}}^{0})}$$

add a term that is negatively proportional to the error between what we have and what we want

$$= Ax - BKx$$
$$= (A - BK)x$$ ← what x does with control

↑ choose K for stability or whatever

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

how to go from y to x?

$\hat{x}(t)$

$u(t) = -K\hat{x}(t)$

$\hat{x}(t+\Delta t) \approx \hat{x}(t) + \Delta t\left(A\hat{x}(t) + Bu(t)\right)$

Apply to state estimation:

$$\dot{\hat{x}} = A\hat{x} + Bu$$

← what $\hat{x}$ <u>should</u> do if our model and our knowledge of initial conditions were <u>perfect</u> (they aren't)

$$\cancel{\dot{\hat{x}} = A\hat{x} + Bu - L(\hat{x} - x)}$$

← add a term that is negatively proportional to error

$$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$$

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$ how to go from y to x?

Apply to state estimation:

$\dot{\hat{x}} = A\hat{x} + Bu, \quad \hat{x}(0) = x_0$ ← what $\hat{x}$ should do if our model and our knowledge of initial conditions were perfect (they aren't)

our estimate

what our estimate should be

$\dot{\hat{x}} = A\hat{x} + Bu \underbrace{- L(\hat{x} - x)}$ ← we can't implement this because we don't know x (that's the point) ... but we do know y

add a term that is negatively proportional to the error

$\dot{\hat{x}} = A\hat{x} + Bu - L(\underbrace{C\hat{x} - y})$

"$\hat{y}$", our best guess at y

error in prediction of y

IMPLEMENTATION

state

input

$$\dot{x} = Ax + Bu$$ ← dynamic model

$$y = Cx$$ ← sensor model

output

controller

$$u = -K\hat{x}$$

$$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$$

observer

RESET $\left\{ \hat{x}(0) = 0 \right.$

RUN
$$\left\{ \begin{array}{l} \vdots \\ u(t) = -K\hat{x}(t) \\ \hat{x}(t+\Delta t) \approx \hat{x}(t) + \Delta t\left(A\hat{x}(t) + Bu(t) - L\left(C\hat{x}(t) - y(t)\right)\right) \\ \vdots \end{array} \right.$$