Tracking

AE353

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We have learned to apply linear state feedback:

$$u = -Kx.$$

When it works (i.e., when all eigenvalues of $A - BK$ have negative real part), linear state feedback does exactly one thing:

$$x(t) \to 0 \quad \text{as} \quad t \to \infty.$$

Equivalently, it makes

$$m(t) \to m_e \quad \text{as} \quad t \to \infty. \qquad \leftarrow \text{remember: } x = m - m_e$$

Since the equilibrium point m_e has to be chosen in advance and cannot be changed, then our controllers cannot (yet) do what we want them to do — for example, make the wheel reach <u>any</u> target angle or make the cat-catching robot reach <u>any</u> target position. For that, we would want

$$m(t) \to m_{des} \quad \text{as} \quad t \to \infty$$

where m_{des} is a desired state that we can change on the fly. We call this <u>tracking</u> (or "reference tracking").

WHAT CAN YOU TRACK

$$\dot{m} = f(m, n)$$ ← It is possible to track a variable m_i if $f(m, n)$ does not depend on m_i

EXAMPLES

$$\begin{bmatrix} \dot{q} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \\ \tau \end{bmatrix}$$ ← what can you track?

q

$$\begin{bmatrix} \dot{q} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \\ \tau - 2\sin(q) \end{bmatrix}$$ ← what can you track?

nothing

$\dot{m} = f(m, n)$

HOW CAN YOU TRACK

① Linearize about m_e, n_e as usual

$\dot{x} = Ax + Bu \qquad x = m - m_e \qquad u = n - n_e$

m_e and n_e must remain constant

② Compute a desired state

CHOOSE

m_{des} ← m_{des} can change on the fly, but must differ from m_e only in those variables that can be tracked

DEFINE

$x_{des} = m_{des} - m_e$

IMPLEMENT

$$u = -K(x - x_{des}) = -K((m - m_e) - (m_{des} - m_e))$$
$$= -K(m - m_{des})$$

WHAT CAN GO WRONG

If the error

$$x - x_{des} = (m - m_e) - (m_{des} - m_e)$$
$$= m - m_{des}$$

is too large, then the input

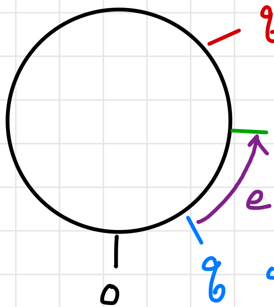
$$u = -K(x - x_{des}) = -K(m - m_{des})$$

might exceed bounds (e.g., maximum torque). We don't include these bounds in our state space model

$$\dot{x} = Ax + Bu$$

and so even if the controller is stable in theory it might not be stable in practice.

ONE WAY TO FIX WHAT GOES WRONG (WHEEL EXAMPLE)



q_{target} target angle

q_{des} desired angle

e_{max} maximum allowable error (you choose this)

q current angle

$$m_{des} = \begin{bmatrix} q_{des} \\ 0 \end{bmatrix}$$

$$q_{des} = \begin{cases} q + e_{max} & \text{if} \quad q + e_{max} < q_{target} \\ q_{target} & \text{if} \quad q - e_{max} < q_{target} < q + e_{max} \\ q - e_{max} & \text{if} \quad q_{target} < q - e_{max} \end{cases}$$

$$q_{des} = \begin{cases} q + e_{max}\left(\dfrac{q_{target} - q}{|q_{target} - q|}\right) & \text{if} \quad |q_{target} - q| > e_{max} \\ q_{target} & \text{otherwise} \end{cases}$$

these are two equivalent ways to implement the same thing