Diagonalization (Part 2)

AE353 Spring ZOZ5 Bretl

## LAST TIME

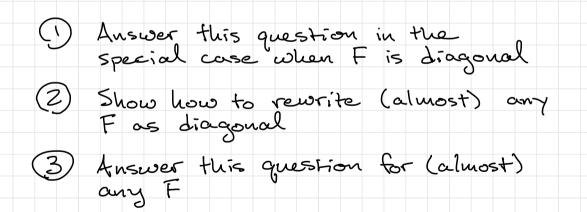
- x = Ax + Bu & state-space model (dynamics) u = - Kx & linear state feedback (controller)

- La X(t) -> O as t-> so if and only if all eigenvalues of A-BK have negative real part

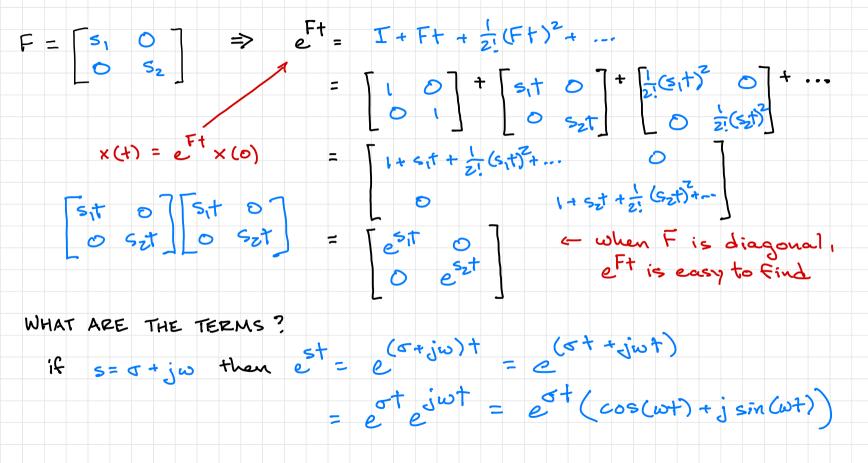
OUR GOLL IS TO PROVE THIS RESULT

## x=Fx < for which F does x(t) > D as t > 00???

STRATEGY

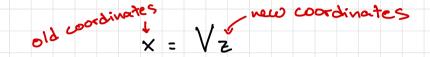


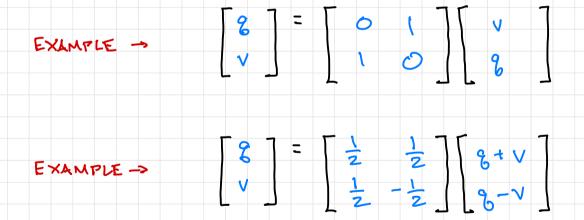
(1) Answer the question in the special case when F is diagonal

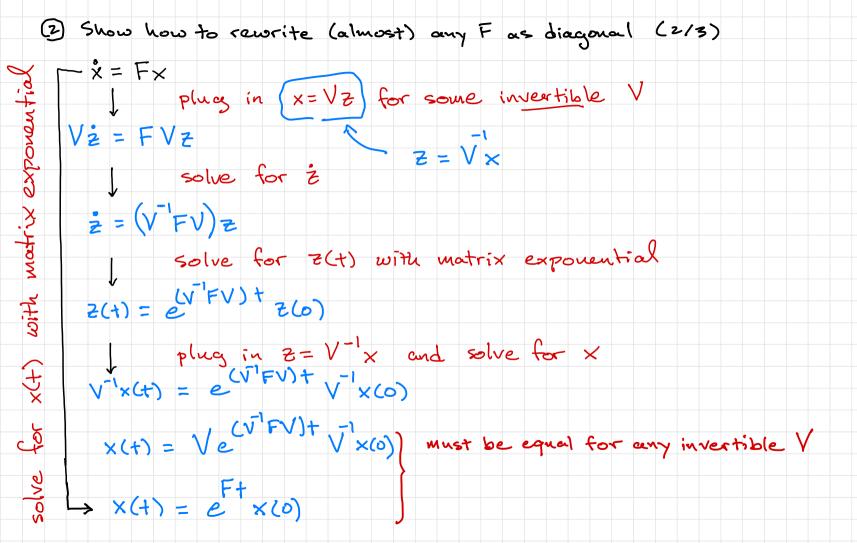


(2) Show how to rewrite (almost) any Fas diagonal (1/3)

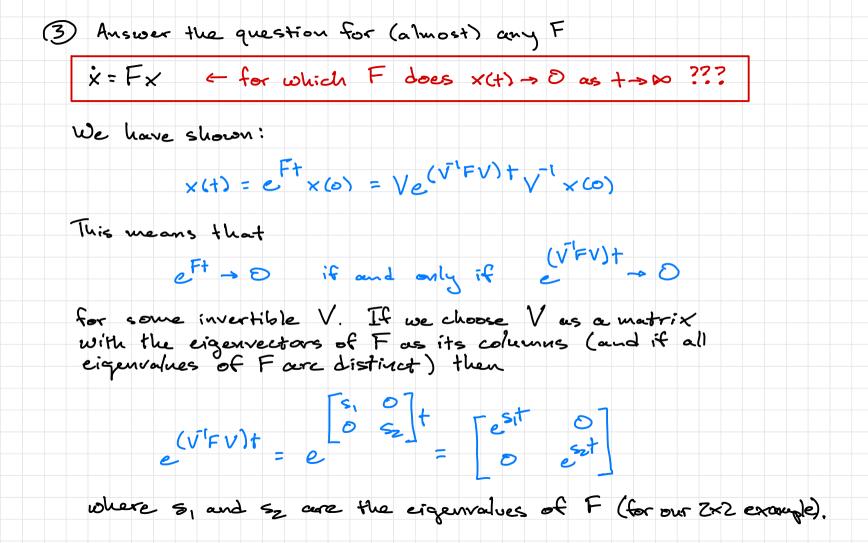
KEY: COORDINATE TRANSFORMATION







(2) Show how to rewrite (almost) any F as diagonal (3/3) $x(t) = e^{Ft} x(o)$ = Ve(VIFV) + VIx(0) for any invertible V choose V so V<sup>-</sup>FV is diagonal  $V F V = \begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix}$   $V = \begin{bmatrix} v_1 & v_2 \end{bmatrix}$  $\Rightarrow FV = V \begin{bmatrix} s_1 & D \\ D & s_2 \end{bmatrix}$  $F_{v_1} = v_1 s_1$  $F[v_1, v_2] = [v_1, v_2][s_1, 0]$  $Fv_2 = v_2 S_2$  $\begin{bmatrix} F_{V_1} & F_{V_2} \end{bmatrix} = \begin{bmatrix} V_1 S_1 & V_2 S_2 \end{bmatrix}$ v, and vz are eigenvectors of F s, and sz are esgenvalues of F



## DEFINITION

The closed-loop system

x = (A-BK)x

is called asymptotically stable if

×(+) = O as + = 00

for any x(0).

## THEOREM

The closed-loop system

× = (A-BK)×

is asymptotically stable if and only if all eigenvalues of

A-BK

have negative real part.