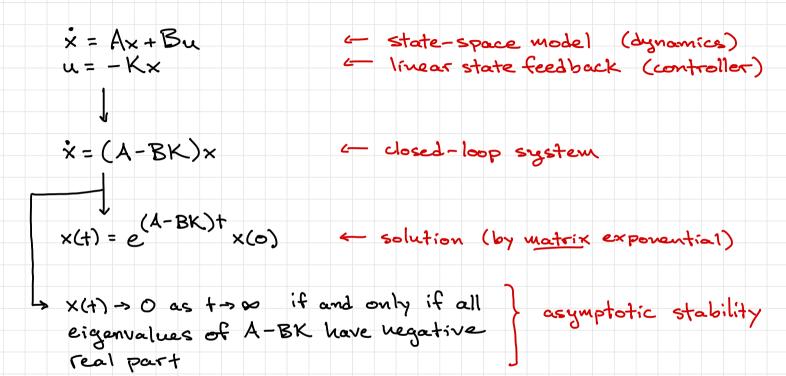
Diagonalization (Part 1)

AE353 Spring 2025 Bret1 LAST TIME



OUR GOAL IS TO PROVE THIS RESULT

x=Fx < for which F does x(+) > 0 as +> > ???

STRATEGY

- (1) Answer this question in the special case when F is diagonal
- (2) Show how to rewrite (almost) any F as diagonal
- 3) Answer this question for (almost)
 any F

(1) Answer the question in the special case when F is diagonal $e^{Ft} = I + Ft + \frac{1}{2!}(Ft)^2 + ...$ $F = \begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix} \Rightarrow e^{t+} = \begin{bmatrix} 1 + Ft + \frac{1}{2!}(Ft)^2 + \dots \\ 0 & s_2 \end{bmatrix} + \begin{bmatrix} 1 + Ft + \frac{1}{2!}(s_1t)^2 \\ 0 & s_2 \end{bmatrix} + \dots$ $x(t) = e^{Ft} \times (0) \qquad = \left[1 + s_1 t + \frac{1}{z_1!} (s_1 t)^2 + \dots \right]$ Sit of Sit of Lossit of Lossit of Country of Sit of WHAT ARE THE TERMS? if $s = \sigma + j\omega$ then $e^{s} = e^{(\sigma + j\omega)} + e^{(\sigma + j\omega)}$ $= e^{\dagger} e^{j\omega t} = e^{\dagger} (\cos(\omega t) + j\sin(\omega t))$

EXAMPLE
$$\rightarrow$$

$$\begin{bmatrix} 8 \\ v \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} + v \end{bmatrix}$$

2) Show how to rewrite (almost) any F as diagonal (2/3) plug in (x=VZ) for some invertible V Vz = FVZ solve for z = (V FV) = solve for Z(t) with matrix exponential Z(+) = e (V FV) + Z(0) plug in Z=V-1x and solve for x V-1x(+) = e (V) FV)+ V-1x(0) x(t) = Ve (VFV)+ -1 V(0)) must be equal for any invertible V