Stability

AE 353
Spring 2025
Bretl

LAST TIME

$$\dot{x} = Ax + Bu \quad \leftarrow \text{state-space model (dynamics)}$$
$$u = -Kx \quad \leftarrow \text{linear state feedback (controller)}$$

$$\downarrow$$

$$\dot{x} = (A - BK)x \quad \leftarrow \text{closed-loop system}$$

$$\downarrow$$

$$x(t) = e^{(A-BK)t}x(0) \quad \leftarrow \text{solution (by matrix exponential)}$$

THIS TIME

Predict what will happen without simulation and without solving for $x(t)$

LOOKED AT EXAMPLES

↓

- what does "good" and "bad" mean?
  - in animation
  - in plots
- how do the terms in $x(t)$ differ for "good" and "bad" results?
- what is the correspondence between the terms in $x(t)$ and the eigenvalues of $A-BK$?

DEFINITION

The closed-loop system

$$\dot{x} = (A - BK)x$$

is called asymptotically stable if

$$x(t) \to 0 \quad \text{as} \quad t \to \infty$$

for any $x(0)$.

THEOREM

The closed-loop system

$$\dot{x} = (A - BK)x$$

is asymptotically stable if and only if all eigenvalues of

$$A - BK$$

have negative real part.