

# Matrix Exponential

AE 353

Spring 2025

Bretl

CAN WE PREDICT WHAT WILL HAPPEN WITHOUT SIMULATION?

open-loop system  $\dot{x} = Ax + Bu$  ← state space model  
controller  $u = -Kx$  ← linear state feedback

closed-loop system  $\dot{x} = Ax + B(-Kx)$   
 $= Ax - BKx$   
 $= (A - BK)x$   
 $\dot{x} = (A - BK)x$   
 $\dot{x} = Mx$

CAN WE PREDICT WHAT WILL HAPPEN WITHOUT SIMULATION?

$$\dot{x} = (a - bk)x \quad \leftarrow \quad x(t) = e^{(a-bk)t} x(0)$$

$$\dot{x} = 3x \quad \leftarrow \quad x(t) = e^{3t} x(0) \quad \leftarrow \quad 3e^{3t} x(0)$$

↑ SCALAR EXPONENTIAL =  $3(e^{3t} x(0))$

$$e^m = 1 + m + \frac{m^2}{2!} + \frac{m^3}{3!} + \dots = \sum_{k=0}^{\infty} \frac{m^k}{k!}$$

$$\dot{x} = (A - BK)x \quad \leftarrow \quad x(t) = e^{(A-BK)t} x(0)$$

↑ MATRIX EXPONENTIAL

$$e^M = I + M + \frac{1}{2!} M^2 + \frac{1}{3!} M^3 + \dots = \sum_{k=0}^{\infty} \frac{1}{k!} M^k$$

CAN WE PREDICT WHAT WILL HAPPEN WITHOUT FINDING  $x(t)$  ?

MONDAY !