

Models

AE353

Spring 2025

Bretl

<https://go.aerospace.illinois.edu/ae353-sp25>

COURSE  
WEBSITE

PLEASE COMPLETE  
HCD SURVEY NOW IF  
YOU HAVE NOT ALREADY

(see "Schedule" page, or  
go to Canvas and click  
on "Assignments")



# MODEL OF DYNAMICS

STEP 0 - get EOMs

$$\boxed{J\ddot{q} = \tau}$$

This is the model we are given. We want to rewrite this model in "state-space form" so it looks like this.  $\dot{x} = Ax + Bu$

STEP 1 - rewrite EOMs as a set of first-order ODEs

$$\ddot{q}$$
$$v = \dot{q}$$

← find time derivative of highest order

← define new variables for each time derivative of lower order

$$J\dot{v} = \tau$$

← rewrite EOMs in terms of new variables

$$\dot{q} = v$$

← add an ODE for each new variable

$$\dot{q} = v$$
$$\dot{v} = (1/J)\tau$$

← collect ODEs together, solving for time derivatives if necessary

$$\begin{bmatrix} \dot{q} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \\ (1/J)\tau \end{bmatrix}$$

← write in standard form

$$\dot{m} = f(m, n) \quad \leftarrow m = \begin{bmatrix} q \\ v \end{bmatrix} \quad n = [\tau]$$

$$\begin{aligned}\dot{g} &= v \\ \dot{v} &= (1/J)\tau\end{aligned}$$

$$\leftrightarrow \dot{x} = Ax + Bu$$

STEP 2 - find an equilibrium point

$$0 = v_e$$

← set time derivatives to zero

$$0 = (1/J)\tau_e$$

$$v_e = 0 \quad \tau_e = 0$$

← solve

$$g_e = \frac{\pi}{2} \quad v_e = 0 \quad \tau_e = 0 \quad \leftarrow \text{pick a solution}$$

$$m_e = \begin{bmatrix} \pi/2 \\ 0 \end{bmatrix} \quad n_e = [0]$$

← write in standard form

STEP 3 - define state and input

$$x = m - m_e = \begin{bmatrix} g \\ v \end{bmatrix} - \begin{bmatrix} g_e \\ v_e \end{bmatrix} = \begin{bmatrix} g - \pi/2 \\ v \end{bmatrix}$$

$$\leftarrow x = m - m_e$$

$$u = n - n_e = [\tau] - [\tau_e] = [\tau]$$

$$\leftarrow u = n - n_e$$

STEP 4 - compute A and B

We defined the state  $x$  and the input  $u$  like this:

$$x = \begin{bmatrix} q - (l/2) \\ v \end{bmatrix} \quad u = [\tau]$$

What matrices A and B would make

$$\dot{x} = Ax + Bu$$

and

$$\begin{bmatrix} \dot{q} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \\ (1/J)\tau \end{bmatrix}$$

describe the same set of ODEs?

$$x = \begin{bmatrix} \quad \\ \quad \end{bmatrix} \quad u = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

MODEL OF CONTROLLER

$$u = -Kx$$

## FOR NEXT TIME

- Setup
- Group preferences form (by 11am Monday)
- PrairieLearn HW1 (by 11am Wednesday)