Bode plot

AE353

Spring 2023

Bretl

WHAT WE SAW

If the desired wheel angle is a sine wave with frequency ω

$$q_{des}(t) = \sin(\omega t)$$

Then the actual wheel angle is also a sine wave with the same frequency ω and with magnitude and angle that depend on ω

$$q(t) = \underbrace{(\text{some transient response})}_{\text{decayed to zero}} + m \sin(\omega t + \theta)$$

EXAMPLES

$\omega = (2\pi/1)$ $\quad m = 0.678$ $\quad \theta = -2.64$

$\omega = (2\pi/2)$ $\quad m = 1.79$ $\quad \theta = -0.723$

$\omega = (2\pi/5)$ $\quad m = 1.09$ $\quad \theta = -0.162$

slower

MODEL (1)

$\dot{x} = Ax + Bu$ dynamic model

$y = Cx$ sensor model

$u = -K(\hat{x} - x_{des})$ controller (w/ tracking)

$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$ observer

$$\dot{x} = Ax + Bu = Ax + B(-K(\hat{x} - x_{des}))$$
$$= Ax - BK\hat{x} + BKx_{des}$$

$$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y) = A\hat{x} - BK(\hat{x} - x_{des}) - LC(\hat{x} - x)$$
$$= LCx + (A - BK - LC)\hat{x} + BKx_{des}$$

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & -BK \\ LC & A-BK-LC \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} BK \\ BK \end{bmatrix} x_{des}$$

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & -BK \\ LC & A-BK-LC \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} BK \\ BK \end{bmatrix} x_{des}$$

MODEL (2)

the state to track: $x_{des} = \begin{bmatrix} q_{des} \\ 0 \end{bmatrix} = \overbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}^{e_1} q_{des}$

INPUT: $e_1^T x_{des} = e_1^T e_1 q_{des} = q_{des}$

OUTPUT: $e_1^T x = q$

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & -BK \\ LC & A-BK-LC \end{bmatrix} \overbrace{\begin{bmatrix} x \\ \hat{x} \end{bmatrix}}^{x_m} + \begin{bmatrix} BKe_1 \\ BKe_1 \end{bmatrix} \begin{bmatrix} q_{des} \end{bmatrix}$$

$$\begin{bmatrix} q \end{bmatrix} = \begin{bmatrix} e_1^T & 0 \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} q_{des} \end{bmatrix}$$

$\dot{x}_m = A_m x_m + B_m u_m$

$y_m = C_m x_m + D_m u_m$

General Result

single input

$$\dot{x}_m = A_m x_m + B_m u_m$$

$$y_m = C_m x_m + D_m u_m$$

single output

transient (decays to zero)

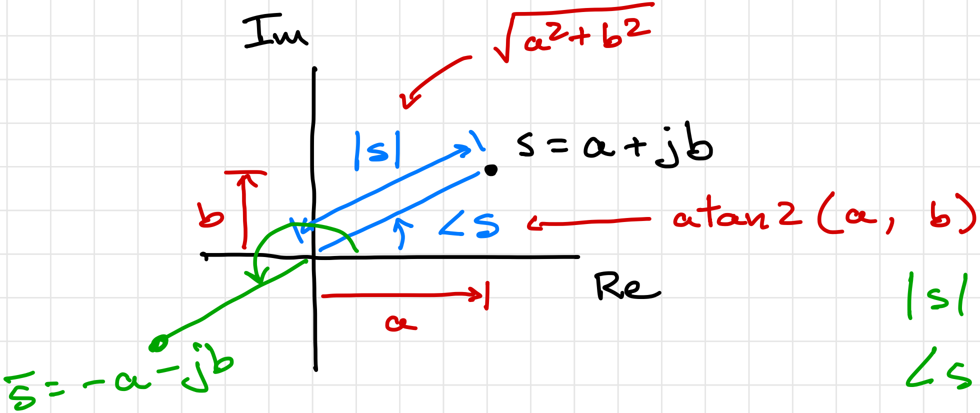
$$u_m(t) = \sin(\omega t) \Longrightarrow y_m(t) = (\cdots) + |H(j\omega)| \sin(\omega t + \angle H(j\omega))$$

$$u_m(t) = \cos(\omega t) \Longrightarrow y_m(t) = (\cdots) + \underbrace{|H(j\omega)|}_{\text{magnitude}} \cos(\omega t + \underbrace{\angle H(j\omega)}_{\text{angle}})$$

a complex number

$$\underbrace{H(s)}_{\text{another complex number}} = C_m(sI - A_m)^{-1} B_m + D_m \longleftarrow \text{TRANSFER FUNCTION}$$

Complex Numbers



$|s|$ = np.absolute(s)

$\angle s$ = np.angle(s)

$$s = |s| e^{j\angle s} = |s|\left(\cos(\angle s) + j\sin(\angle s)\right)$$

$$\frac{1}{2j}\left(e^{j\omega t} - e^{-j\omega t}\right) = \frac{1}{2j}\Big(\left(\cos(\omega t) + j\sin(\omega t)\right) - \left(\cos(-\omega t) + j\sin(-\omega t)\right)\Big)$$

$$= \frac{1}{2j}\Big(\left(\cancel{\cos\omega t} + \underline{j\sin\omega t}\right) - \left(\cancel{\cos\omega t} - \underline{j\sin\omega t}\right)\Big)$$

$$= \frac{1}{2j}\left(2j\sin\omega t\right)$$

$$= \sin\omega t$$

$$q_{des}(t) \approx \sum_{k=0}^{\infty} a_k \sin(2k\pi t)$$

Converting to/from "decibels" (dB)

absolute		dB
m	$\longrightarrow$	$20 \log_{10} m$
$10^{(\tilde{m}/20)}$	$\longleftarrow$	$\tilde{m}$

Bandwidth