

Transfer function

AE 353

Spring 2023

Bretl

WHAT WE SAW

If the desired wheel angle is a sine wave with frequency ω

$$\theta_{des}(t) = \sin(\omega t)$$

Then the actual wheel angle is also a sine wave with the same frequency ω and with magnitude and angle that depend on ω

$$\theta(t) = \underbrace{(\text{some transient response})}_{\text{decayed to zero}} + m \sin(\omega t + \Theta)$$

EXAMPLES

$$\omega = (2\pi/1)$$

$$m = 0.678$$

$$\Theta = -2.64$$

$$\omega = (2\pi/2)$$

$$m = 1.79$$

$$\Theta = -0.723$$

$$\omega = (2\pi/5)$$

$$m = 1.09$$

$$\Theta = -0.162$$

↓ slower

$$\begin{aligned}\dot{x} &= Ax + Bu && \text{dynamic model} \\ y &= Cx && \text{sensor model}\end{aligned}$$

MODEL (1)

$$\begin{aligned}u &= -K(\hat{x} - x_{des}) && \text{controller (w/ tracking)} \\ \dot{\hat{x}} &= A\hat{x} + Bu - L(C\hat{x} - y) && \text{observer}\end{aligned}$$

$$\begin{aligned}\dot{x} &= Ax + Bu = Ax + B(-K(\hat{x} - x_{des})) \\ &= Ax - BK\hat{x} + BKx_{des}\end{aligned}$$

$$\begin{aligned}\dot{\hat{x}} &= A\hat{x} + Bu - L(C\hat{x} - y) = A\hat{x} - BK(\hat{x} - x_{des}) - LC(\hat{x} - x) \\ &= LCx + (A - BK - LC)\hat{x} + BKx_{des}\end{aligned}$$

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & -BK \\ LC & A - BK - LC \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} BK \\ BK \end{bmatrix} x_{des}$$

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & -BK \\ LC & A-BK-LC \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} BK \\ BK \end{bmatrix} x_{des}$$

MODEL (2)

the state to track: $x_{des} = \begin{bmatrix} q_{des} \\ 0 \end{bmatrix} = \overbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}^{e_1} q_{des}$

INPUT: $e_1^T x_{des} = e_1^T e_1 q_{des} = q_{des}$

OUTPUT: $e_1^T x = q$

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \underbrace{\begin{bmatrix} A & -BK \\ LC & A-BK-LC \end{bmatrix}}_{A_m} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \underbrace{\begin{bmatrix} BK e_1 \\ BK e_1 \end{bmatrix}}_{B_m} \underbrace{\begin{bmatrix} q_{des} \end{bmatrix}}_{u_m}$$

$$\underbrace{\begin{bmatrix} q \end{bmatrix}}_{y_m} = \underbrace{\begin{bmatrix} e_1^T & 0 \end{bmatrix}}_{C_m} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D_m} \underbrace{\begin{bmatrix} q_{des} \end{bmatrix}}_{u_m}$$

$$\begin{aligned} \dot{x}_m &= A_m x_m + B_m u_m \\ y_m &= C_m x_m + D_m u_m \end{aligned}$$

GENERAL RESULT

$$\dot{x}_m = A_m x_m + B_m u_m$$

single input

$$y_m = C_m x_m + D_m u_m$$

single output

transient
(decays to zero)

$$u_m(t) = \sin(\omega t) \Rightarrow y_m(t) = (\dots) + |H(j\omega)| \sin(\omega t + \angle H(j\omega))$$

$$u_m(t) = \cos(\omega t) \Rightarrow y_m(t) = (\dots) + \underbrace{|H(j\omega)|}_{\text{magnitude}} \cos(\omega t + \underbrace{\angle H(j\omega)}_{\text{angle}})$$

a complex number

$$H(s) = C_m (sI - A_m)^{-1} B_m + D_m$$

another complex number

TRANSFER FUNCTION