More observer design

AE353
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Bretl

state, input

$$\dot{x} = Ax + Bu \quad \leftarrow \text{dynamic model}$$

$$y = Cx \quad \leftarrow \text{sensor model}$$

output

controller

$$u = -K\hat{x}$$

$$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$$

observer

IMPLEMENTATION

$$\hat{x}(t+\Delta t) \approx \hat{x}(t) + \Delta t\left(A\hat{x}(t) + Bu(t) - L(C\hat{x}(t) - y(t))\right)$$

ANALYSIS

$$\dot{x}_{err} = (A - LC)\, x_{err} \quad \text{where} \quad x_{err} = \hat{x} - x$$

$\Rightarrow$ ① $x_{err}(t) \to 0$ as $t \to \infty$ if all eigenvalues of $A-LC$ have negative real part

② $x_{err}(t) = e^{(A-LC)t}\, x_{err}(0)$

FACTS

$$(M+N)^T = M^T + N^T$$
$$(MN)^T = N^T M^T$$

HOW TO CHOOSE L?

$$\dot{x} = (A - BK)x$$

$$\dot{x}_{err} = (A - LC)x_{err}$$

$$0 = \det(sI - (A - BK))$$

$$K = \text{place-poles}(A, B, p)$$

$$0 = \det(sI - (A - LC))$$
$$= \det((sI - (A - LC))^T)$$
$$= \det((sI)^T - (A - LC)^T)$$
$$= \det(sI - (A^T - (LC)^T))$$
$$= \det(sI - (A^T - C^T L^T))$$

$$L = \text{place\_poles}(A^T, C^T, p)^T$$

WHEN IS OBSERVER DESIGN POSSIBLE?

$\dot{x} = (A - BK)x$

controllable when

$W_c = [B \;\; AB \;\; A^2B \;\; \cdots \;\; A^{n-1}B]$

is full rank

$\dot{x}_{err} = (A - LC)x_{err}$

observable when

$[C^T \;\; A^TC^T \;\; (A^T)^2C^T \;\; \cdots \;\; (A^T)^{n-1}C^T]$

is full rank

$$W_o = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

DOES THE CONTROLLER STILL WORK?

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

sensor model

dynamic model

$$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$$

observer

$$u = -K\hat{x}$$

controller

- does this screw anything up?
- must we change our choice of K?

$$\dot{x} = (A-BK)x$$

is NOT the closed-loop system anymore!

DOES THE CONTROLLER STILL WORK?

$\dot{x} = Ax + Bu$

$y = Cx$

dynamic model

sensor model

$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$

observer

$u = -K\hat{x}$

controller

We had found

$$\dot{x}_{err} = (A - LC)\, x_{err} \qquad \text{where} \qquad x_{err} = \hat{x} - x.$$

Now, let's find

$$\dot{x} =$$

Write in matrix form:

$$\begin{bmatrix} \dot{x} \\ \dot{x}_{err} \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \begin{bmatrix} x \\ x_{err} \end{bmatrix}$$

DOES THE CONTROLLER STILL WORK?

$$\dot{x} = Ax + Bu \qquad \dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y) \qquad u = -K\hat{x}$$
$$y = Cx$$

dynamic model

sensor model

observer

controller

$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{x}_{err} \end{bmatrix} = \begin{bmatrix} A-BK & -BK \\ 0 & A-LC \end{bmatrix}}_{\text{stability depends on the eigenvalues of this matrix}} \begin{bmatrix} x \\ x_{err} \end{bmatrix}$$

← closed-loop system

FACT:

$$\det\left(\begin{bmatrix} F & H \\ 0 & G \end{bmatrix}\right) = \det(F)\det(G)$$

CONSEQUENCE:

the eigs of $\begin{bmatrix} A-BK & -BK \\ 0 & A-LC \end{bmatrix}$

are the <u>union</u> of the eigs of $A-BK$ and of $A-LC$

so... choose K <u>as if</u> $\dot{x} = (A-BK)x$ <u>were</u> the closed-loop system — this is still OK!

SEPARATION PRINCIPLE

you can design the observer and the controller separately

1. design the observer while ignoring the controller (works for arbitrary u)

2. design the controller assuming the state estimate is perfect

NONLINEAR SENSOR MODELS

$$\underset{\dot{x}}{\dot{m}} = f(m,n) \approx \cancel{f(m_e, n_e)}^{\,\phi} + \underbrace{\left.\frac{\partial f}{\partial m}\right|_{(m_e,n_e)}}_{A} \overbrace{(m - m_e)}^{x} + \underbrace{\left.\frac{\partial f}{\partial n}\right|_{(m_e,n_e)}}_{B} \overbrace{(n - n_e)}^{u}$$

$$o = g(m,n) \approx g(m_e, n_e) + \left.\frac{\partial g}{\partial m}\right|_{(m_e,n_e)} (m - m_e) + \left.\frac{\partial g}{\partial n}\right|_{(m_e,n_e)} (n - n_e)$$

$$\underbrace{o - \overbrace{g(m_e, n_e)}^{o_e}}_{y} \approx \underbrace{\left.\frac{\partial g}{\partial m}\right|_{(m_e,n_e)}}_{C} \overbrace{(m - m_e)}^{x} + \underbrace{\left.\frac{\partial g}{\partial n}\right|_{(m_e,n_e)}}_{D} \overbrace{(n - n_e)}^{u}$$