Observer design

AE353

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state

input

$$\dot{x} = Ax + Bu$$ ← dynamic model

$$y = Cx$$ ← sensor model

output

$$u = -K\hat{x}$$ ← controller

$$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$$ ← observer

HOW TO IMPLEMENT IT ?

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

MODEL

$u = -K\hat{x}$ ← CONTROLLER

$\dot{\hat{x}} = \underline{A\hat{x} + Bu - L(C\hat{x} - y)}$ ← OBSERVER

RESET $\left\{ \hat{x}(0) = 0 \right.$

$\vdots$

RUN $\left\{ \begin{array}{l} u(t) = -K\hat{x}(t) \\ \underline{\hat{x}(t+\Delta t)} \approx \underline{\hat{x}(t)} + \underline{\Delta t}\left(\underline{A\hat{x}(t) + Bu(t) - L(C\hat{x}(t) - y(t))}\right) \end{array} \right.$

$\vdots$

WHY DOES IT MAKE SENSE?

$$\begin{rcases} \dot{x} = Ax + Bu \\ y = Cx \end{rcases}$$ how to go from y to x?

Take inspiration from state feedback:

$\dot{x} = Ax$ ← what x does without control

$\dot{x} = Ax - BK(\overbrace{x - 0}^{\text{error}})$ ← add a term that is negatively proportional to error

$= Ax - BKx$

$= (A - BK)x$ ← what x does with control

WHY DOES IT MAKE SENSE?

$$\left.\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}\right\}$$ how to go from y to x?

Apply to state estimation:

$$\dot{\hat{x}} = A\hat{x} + Bu$$

← what $\hat{x}$ should do if our model and our knowledge of initial conditions were perfect (they aren't)

$$\dot{\hat{x}} = \cancel{A\hat{x} + Bu - L(\hat{x} - x)}$$

← add a term that is negatively proportional to error

$$\dot{\hat{x}} = A\hat{x} + Bu - L(\underbrace{C\hat{x}}_{\hat{y}} - y)$$

WHEN DOES IT WORK?

$$\dot{x} = Ax + Bu \qquad u = -K\hat{x}$$
$$y = Cx \qquad \dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$$

$$x_{err} = \hat{x} - x$$ ← does this converge to zero or not?

$$\dot{x}_{err} = \dot{\hat{x}} - \dot{x}$$
$$= A\hat{x} + \cancel{Bu} - L(C\hat{x} - y) - Ax - \cancel{Bu}$$
$$= A(\hat{x} - x) - L(C\hat{x} - Cx)$$
$$= A(\hat{x} - x) - LC(\hat{x} - x)$$
$$= A x_{err} - LC x_{err}$$

$$\boxed{\dot{x}_{err} = (A - LC)\, x_{err}}$$

$$\dot{x} = (A - BK)x$$