

LQR (problem statement)

AE 353

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Bretl

Linear Quadratic Regulator (LQR)

minimize $u[t_0, \infty), x[t_0, \infty)$

subject to

$$\int_{t_0}^{\infty} (x(t)^T Q x(t) + u(t)^T R u(t)) dt$$
$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$x(t_0) = x_0$$

total cost

cost

constraints

decision variables

The minimizer (i.e., the input that achieves minimum cost) is

$$u(t) = -K x(t)$$

and the minimum (i.e., the minimum cost) is

$$x_0^T P x_0$$

where K and P can be found in python as follows:

```
def lqr(A, B, Q, R):  
    P = linalg.solve_continuous_are(A, B, Q, R)  
    K = linalg.inv(R) @ B.T @ P  
    return K, P
```

WEIGHTS