LQR (error vs effort)

AE353

Spring 2023

Bretl

$$\dot{x} = Ax + Bu$$
$$u = -Kx$$

HOW TO FIND K ?

1. Gain tuning (i.e., guess and check)
   - make a small change to K
   - check if all eigenvalues of A-BK have negative real part
   - repeat until satisfied

2. Eigenvalue placement
   - choose desired eigenvalue locations
   - apply "place_poles" or Ackermann's method

3. LQR (minimize a cost)
   - choose weights on the cost of non-zero x and u
   - choose K to minimize total, integrated cost

$$\dot{x} = [5]x + [1]u$$

$$u = -[k]x$$

$$\dot{x} = [5-k]x$$

$s = 5-k$

$$x(t) = e^{(5-k)t}\,x(0)$$

Linear Quadratic Regulator (LQR)

$$\underset{u_{[t_0,\infty)}}{\text{minimize}} \quad \int_{t_0}^{\infty} \left(x(t)^T Q\, x(t) + u(t)^T R\, u(t) \right) dt$$

$$\text{subject to} \quad \dot{x}(t) = A x(t) + B u(t)$$
$$x(t_0) = x_0$$

The minimizer (i.e., the input that achieves minimum cost) is

$$u(t) = -K\, x(t)$$

and the minimum (i.e., the minimum cost) is

$$x_0^T P x_0$$

where K and P can be found in python as follows:

```
def lqr(A, B, Q, R):
    P = linalg.solve_continuous_are(A, B, Q, R)
    K = linalg.inv(R) @  B.T @ P
    return K, P
```