Controllability

AE353

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Bretl

ACKERMANN'S METHOD

- Compute the characteristic equation that we want:

$$(s-p_1)\cdots(s-p_n) = s^n + r_1 s^{n-1} + \cdots + r_{n-1}s + r_n$$

$$(s+2)(s+3) = s^2 + 5s + 6$$

- Compute the characteristic equation that we have:

$$\det(sI - A) = s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n$$

- Compute the controllability matrix of the original system (and check that $\det(W) \neq 0$):

$$W = \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix}$$

- Compute the controllability matrix of the transformed system:

$$W_{\text{ccf}} = \begin{bmatrix} B_{\text{ccf}} & A_{\text{ccf}}B_{\text{ccf}} & \cdots & A_{\text{ccf}}^{n-1}B_{\text{ccf}} \end{bmatrix}$$

$$\dot{z} = \underbrace{V^{-1}AV}_{A_{ccf}}z + \underbrace{V^{-1}B}_{B_{ccf}}u$$

where

$$A_{\text{ccf}} = \begin{bmatrix} -a_1 & -a_2 & \cdots & -a_{n-1} & -a_n \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \qquad B_{\text{ccf}} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\dot{x} = Ax + Bu$$

$$x = Vz$$

$$\Leftrightarrow \quad z = V^{-1}x$$

- Compute the gains for the transformed system:

$$K_{\text{ccf}} = \begin{bmatrix} r_1 - a_1 & \cdots & r_n - a_n \end{bmatrix}$$

- Compute the gains for the original system:

$$K = K_{\text{ccf}}\overbrace{W_{\text{ccf}}W^{-1}}^{V^{-1}}$$

The system

$$\dot{x} = Ax + Bu$$

is controllable if

$$W = [B \quad AB \quad \cdots \quad A^{n-1}B]$$

"n" is the number of states (i.e., the size of x)

has full rank.

WHEEL

$$A = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$W = \begin{bmatrix} B & AB \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$W = \begin{bmatrix} B & AB & A^2B & \dots & A^{n-1}B \end{bmatrix}$$

$$\dot{x} = A x + B u$$

2 1

nonlinear input

$$\dot{m} = f(m, n)$$

nonlinear state