Eigenvalue placement

AE353

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$\dot{m} = f(m, n)$

↓ linearize about equilibrium point m_e, n_e

$\dot{x} = Ax + Bu$ where $x = m - m_e$ and $u = n - n_e$

↓ apply linear state feedback $u = -Kx$

$\dot{x} = \boxed{(A - BK)}\, x$

↑ asymptotically stable if all eigenvalues of this matrix have negative real part

The eigenvalues of a matrix are the roots of its characteristic polynomial

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$u = -\begin{bmatrix} 8 & 5 \end{bmatrix} x$$

$$A - BK = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 8 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 8 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$$

$$\det(sI - (A - BK))$$

$$= \det\left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \right) = \det\left(\begin{bmatrix} s & -1 \\ 6 & s+5 \end{bmatrix} \right)$$

$$= s(s+5) - (-1)(6)$$

$$= s^2 + 5s + 6 = (s+2)(s+3) = 0$$

$\hookrightarrow$ $s_1 = -2$ $\quad s_2 = -3$

One way to place eigenvalues is to equate coefficients of the characteristic polynomial

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$u = -\begin{bmatrix} k_1 & k_2 \end{bmatrix} x$$

← find k_1 and k_2 to put closed-loop eigenvalues at -1 and -4

What do we want?

$$(s+1)(s+4) = \boxed{s^2 + 5s + 4}$$

$k_1 = 6$

$k_2 = 5$

What do we have?

$$A - BK = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2-k_1 & -k_2 \end{bmatrix}$$

$$\det(sI - (A-BK)) = \det\left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 2-k_1 & -k_2 \end{bmatrix} \right) = \det\left(\begin{bmatrix} s & -1 \\ k_1-2 & s+k_2 \end{bmatrix} \right)$$

$$= s(s+k_2) - (-1)(k_1-2) = \boxed{s^2 + k_2 s + (k_1-2)}$$

It is possible to automate the process of eigenvalue placement

See python demo