

Eigenvalue placement

AE 353

Spring 2023

Bretl

$$\dot{m} = f(m, n)$$



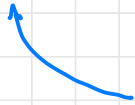
linearize about equilibrium point m_e, n_e

$$\dot{x} = Ax + Bu \quad \text{where } x = m - m_e \text{ and } u = n - n_e$$



apply linear state feedback $u = -Kx$

$$\dot{x} = \boxed{(A - BK)}x$$



asymptotically stable if all eigenvalues of this matrix have negative real part

The eigenvalues of a matrix are the roots of its characteristic polynomial

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$u = -\begin{bmatrix} 8 & 5 \end{bmatrix} x$$

$$A - BK = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 8 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 8 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$$

$$\det(sI - (A - BK))$$

$$= \det \left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \right) = \det \left(\begin{bmatrix} s & -1 \\ 6 & s+5 \end{bmatrix} \right)$$

$$= s(s+5) - (-1)(6)$$

$$= s^2 + 5s + 6 = (s+2)(s+3) = 0$$

$$\hookrightarrow s_1 = -2 \quad s_2 = -3$$

One way to place eigenvalues is to equate coefficients of the characteristic polynomial

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$u = -[k_1 \ k_2] x$$

← find k_1 and k_2 to put closed-loop eigenvalues at -1 and -4

What do we want?

$$(s+1)(s+4) = \boxed{s^2 + 5s + 4}$$

$$k_1 = 6$$

$$k_2 = 5$$

What do we have?

$$A - BK = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} [k_1 \ k_2] = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2-k_1 & -k_2 \end{bmatrix}$$

$$\det(sI - (A - BK)) = \det\left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 2-k_1 & -k_2 \end{bmatrix}\right) = \det\left(\begin{bmatrix} s & -1 \\ k_1-2 & s+k_2 \end{bmatrix}\right)$$

$$= s(s+k_2) - (-1)(k_1-2) = \boxed{s^2 + k_2s + (k_1-2)}$$

It is possible to automate the process of eigenvalue placement

See python demo

