Linearization

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AE 353
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Bretl

$$\dot{x} = \underset{\substack{\uparrow \\ \text{state}}}{A x} + B \underset{\substack{\uparrow \\ \text{input}}}{u}$$

← state space model

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ -1 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1/2 \end{bmatrix} \begin{bmatrix} u_1 \end{bmatrix}$$

WHEEL

$$(J + mr^2)\ddot{q}_b = \tau - mgr\sin q_b \quad \leftarrow \quad c_1\ddot{q}_b = \tau - c_2 \sin q_b$$

STEP 1 – rewrite as a set of first-order ODEs

$\ddot{q}_b$ ← find time derivative of q_b of highest order

$v = \dot{q}_b$ ← define new variables for each time derivative of q with lower order

$c_1\dot{v} = \tau - c_2 \sin q_b$ ← rewrite ODE in terms of new variables

$\dot{q}_b = v$ ← add an ODE for each lower-order derivative of q_b

$$\dot{q} = v$$
$$\dot{v} = (\tau - c_2 \sin q_b)/c_1$$

← collect ODEs together, solving for time derivatives if necessary

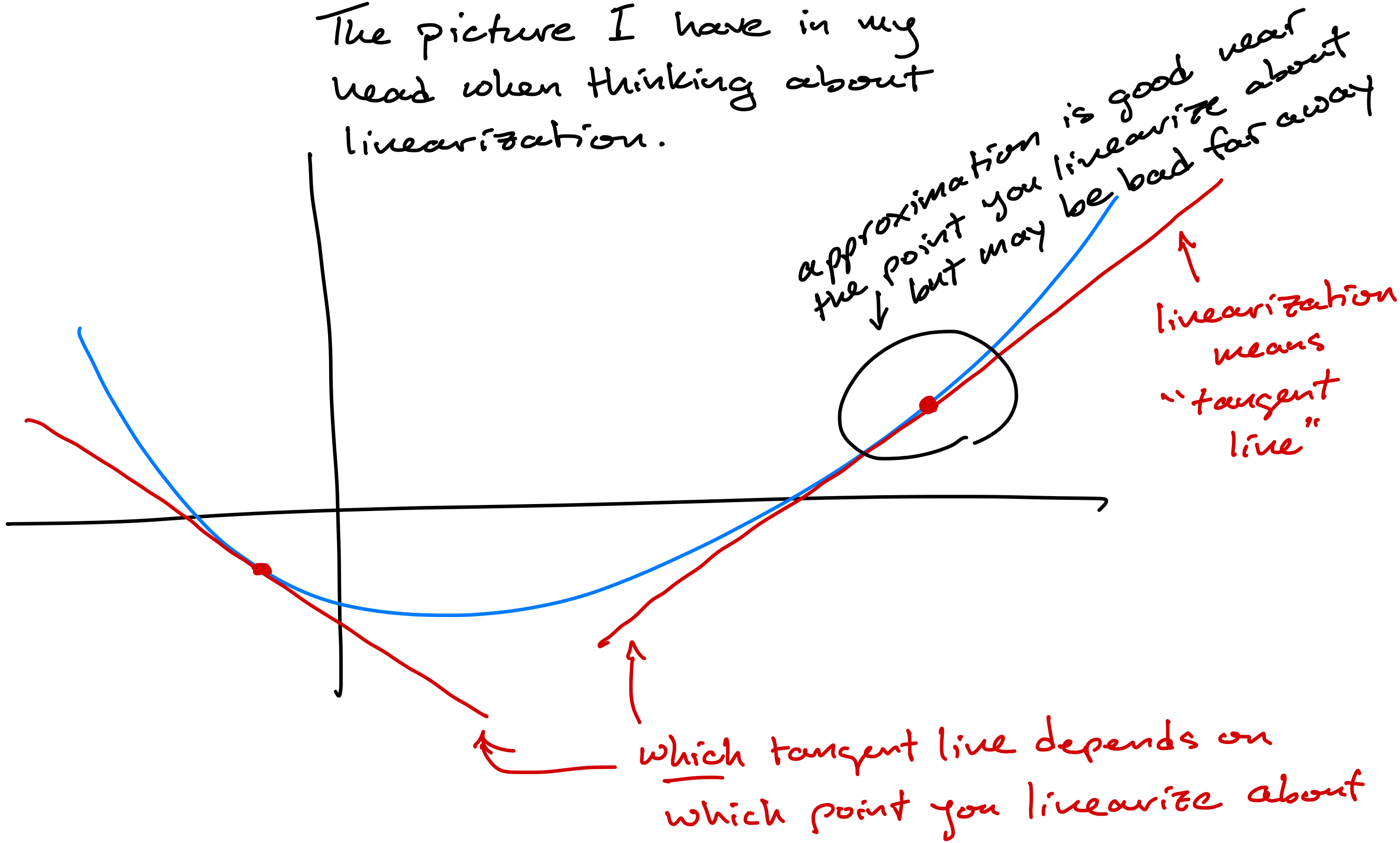
$\dot{m}$

← write in standard form as $\boxed{\dot{m} = f(m, n)}$

$$\begin{bmatrix} \dot{q}_b \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \\ (\tau - c_2 \sin q_b)/c_1 \end{bmatrix}$$

$$m = \begin{bmatrix} q_b \\ v \end{bmatrix} \qquad n = \begin{bmatrix} \tau \end{bmatrix}$$

The picture I have in my head when thinking about linearization.



$$\begin{bmatrix} \dot{q}_6 \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \\ (\tau - c_2 \sin q_6)/c_1 \end{bmatrix}$$

STEP 2 - find an equilibrium point

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} v_e \\ (\tau_e - c_2 \sin q_{6e})/c_1 \end{bmatrix}$$ ← set time derivatives to zero

$$v_e = 0$$
$$\tau_e = c_2 \sin q_{6e}$$
← solve

$q_{6e} = \pi$ $\quad \tau_e = 0$ ← pick a solution
$v_e = 0$

← write in standard form as m_e and n_e

$$\underbrace{\begin{bmatrix} q_{6e} \\ v_e \end{bmatrix} = \begin{bmatrix} \pi \\ 0 \end{bmatrix}}_{m_e} \qquad \underbrace{[\tau_e] = [0]}_{n_e}$$

STEP 3 - define state and input

$$x = \begin{bmatrix} q - \pi \\ v \end{bmatrix} \qquad \leftarrow \ x = m - m_e$$

$$u = \begin{bmatrix} \tau \end{bmatrix} \qquad \leftarrow \ u = n - n_e$$

$$\begin{bmatrix} \dot q \\ \dot v \end{bmatrix} = \begin{bmatrix} v \\ (\tau - c_2 \sin q)/c_1 \end{bmatrix} \qquad \begin{bmatrix} q_e \\ v_e \end{bmatrix} = \begin{bmatrix} \pi \\ 0 \end{bmatrix} \qquad [\tau_e] = [0]$$

STEP 4 - compute A and B

$$\dot m = f(m,n)$$

$$\approx \cancelto{0}{f(m_e,n_e)} + \underbrace{\frac{\partial f}{\partial m}\Big|_{m_e,n_e}}_{A} \underbrace{(m - m_e)}_{x} + \underbrace{\frac{\partial f}{\partial n}\Big|_{m_e,n_e}}_{B} \underbrace{(n - n_e)}_{u}$$

$$\dot x =$$

$$A = \begin{bmatrix} 0 & 1 \\ -(c_2 \cos q_e)/c_1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ c_2/c_1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1/c_1 \end{bmatrix}$$