Day 34

Application of frequency response analysis to the drone

AE353

Spring 2022

Bretl

33.41

35.329 32.96 39.72

35.997 33.41

ω	MAG	ANG
1	1.12	-0.45
2	1.22	-1.34
5	0.22	-2.76

7.142

8.394

6.593

$$\dot{x} = Ax + Bu$$
$$u = u_{des} - K(x - x_{des})$$

$$x_{des} = \begin{bmatrix} q_{des} \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{M} \underbrace{[q_{des}]}_{r}$$

$$\dot{x} = Ax + B(u_{des} - K(x - Mr))$$
$$= (A - BK)x + Bu_{des} + BKMr$$

$$0 = Ax_{des} + Bu_{des}$$

$$-Ax_{des} = -AMr$$

$$\dot{x} = \underbrace{(A - BK)}_{A_m} x \underbrace{- (A - BK)M}_{B_m} r$$
$$y = \underbrace{M^T}_{C_m} x$$

$$\dot{x}_m = A_m x_m + B_m u_m$$
$$y_m = C_m x_m$$

$$\dot{x}_m = A_m x_m + B_m u_m$$

$$y_m = C_m x_m$$

General Result

transient (decays to zero)

s

$$u_m(t) = \frac{1}{2}\sin(\omega t) \Rightarrow y_m(t) = (\cdots) + \frac{1}{2}\left|H(j\omega)\right| \sin\left(\omega t + \angle H(j\omega)\right)$$

$$u_m(t) = \cos(\omega t) \Rightarrow y_m(t) = (\cdots) + \left|H(j\omega)\right| \cos\left(\omega t + \angle H(j\omega)\right)$$

magnitude

angle

a complex number

$$H(s) = C_m(sI - A_m)^{-1}B_m$$

← TRANSFER FUNCTION

another complex number

$$\dot{x} = Ax + Bu \qquad u = u_{des} - K(\hat{x} - x_{des})$$
$$y = Cx \qquad \dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$$

$$\dot{x} = Ax + B(u_{des} - K(\hat{x} - x_{des}))$$
$$= Ax + Bu_{des} - BK(x_{err} + x) + BKx_{des}$$
$$= (A - BK)x - BKx_{err} + Bu_{des} + BKx_{des}$$

$x_{err} = \hat{x} - x$

$\Rightarrow \hat{x} = x_{err} + x$

$0 = Ax_{des} + Bu_{des}$

$$= (A - BK)x - BKx_{err} - (A - BK)x_{des}$$
$$= (A - BK)x - BKx_{err} - (A - BK)Mr$$

$r = [p_{y_{des}}]$

M_r

$M = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$

$$\dot{x}_{err} = (A - LC)x_{err}$$

$$\begin{bmatrix} \dot{x} \\ \dot{x}_{err} \end{bmatrix} = \begin{bmatrix} A - BK & -BK \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x \\ x_{err} \end{bmatrix} + \begin{bmatrix} -(A - BK)M \\ 0 \end{bmatrix} r$$

$$\begin{bmatrix} p_y \end{bmatrix} = \begin{bmatrix} M^T & 0 \end{bmatrix} \begin{bmatrix} x \\ x_{err} \end{bmatrix}$$

$$\dot{x}_m = A_m x_m + B_m u_m$$
$$y_m = C_m x_m$$

Converting to/from "decibels" (dB)

absolute		dB
m	$\longrightarrow$	$20 \log_{10} m$
$10^{(\tilde{m}/20)}$	$\longleftarrow$	$\tilde{m}$