Day 25

Optimal observers

AE353

Spring 2022

Bretl

$\dot{x} = Ax + Bu$
$y = Cx$

← n_x number of states
n_u number of inputs
n_y number of outputs

OPTIMAL CONTROLLER

$u = -Kx$ where $K = lqr(A, B, Q_c, R_c)$

diagonal w/ positive numbers
$\left\{ \begin{array}{l} Q_c \text{ is } n_x \times n_x \quad \leftarrow \text{error} \\ R_c \text{ is } n_u \times n_u \quad \leftarrow \text{effort} \end{array} \right.$

OPTIMAL OBSERVER

$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$ where $L = lqr(A^T, C^T, R_o^{-1}, Q_o^{-1})^T$

diagonal w/ positive numbers
$\left\{ \begin{array}{l} Q_o \text{ is } n_y \times n_y \quad \leftarrow \text{sensors} \\ R_o \text{ is } n_x \times n_x \quad \leftarrow \text{dynamics} \end{array} \right.$

height of quadrotor

$y = x$

$y = 1$

$\hat{x} = 1$

$y_1 = x + n_1$

$y_2 = x + n_2$

$y_1 = 1$

$y_2 = 2$

$\hat{x} = \cancel{1.5}$

$\cancel{2.0}$

1.8 ?

1.9 ?

$\hat{x}$	n_1	n_2
1.5	−0.5	0.5
2.0	−1.0	0.0
1.8	−0.8	0.2

$$\underset{x}{\text{minimize}} \quad q_1(y_1 - x)^2 + q_2(y_2 - x)^2$$

$$\underset{u_{[t_0,\infty)}}{\text{minimize}} \quad \int_{t_0}^{\infty} \Big(\underbrace{x(t)^T Q_c\, x(t)}_{\text{error}} + \underbrace{u(t)^T R_c\, u(t)}_{\text{effort}} \Big)\, dt$$

$$\boxed{u = -K_c x}$$

subject to

$$\dot{x}(t) = A x(t) + B u(t) \quad \text{for } t \in [t_0, \infty)$$

$$x(t_0) = x_0$$

OPTIMAL CONTROLLER

OPTIMAL OBSERVER

$$\underset{x(t_1),\, n_{(-\infty,t_1]},\, d_{(-\infty,t_1]}}{\text{minimize}} \quad \int_{-\infty}^{t_1} \Big(\underbrace{n(t)^T Q_o\, n(t)}_{\text{sensor noise}} + \underbrace{d(t)^T R_o\, d(t)}_{\text{process disturbance}} \Big)\, dt$$

subject to

$$\left.\begin{aligned} \dot{x}(t) &= A x(t) + B u(t) + d(t) \\ y(t) &= C x(t) + n(t) \end{aligned}\right\} \text{ for } t \in (-\infty, t_1]$$

$$\boxed{\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)}$$

$$d(t) = \dot{x}(t) - \big(A x(t) + B u(t)\big)$$

$$-n(t) = C x(t) - y(t)$$

$$\left.\begin{aligned} &\text{if } t_1 = t_\alpha \text{ then } x(t_1) = \alpha \\ &\text{if } t_1 = t_\beta \text{ then } x(t_1) = \beta \end{aligned}\right\}$$

integrate this starting at

$$\hat{x}(t_\alpha) = \alpha$$

then the solution is

$$\hat{x}(t_\beta) = \beta$$