Day 22

more about...

Observers (implementation, design, and analysis)

AE353

Spring 2022

Bretl

state

input

$$\boxed{\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}}$$

← dynamic model

← sensor model

output

$\cancel{-L(\hat{x} - x)}$

$$\boxed{\begin{aligned} u &= -K\hat{x} \\ \dot{\hat{x}} &= A\hat{x} + Bu - \overbrace{L(C\hat{x} - y)} \end{aligned}}$$

← controller

← observer

EXAMPLE (control of platform angle, no gravity)

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad x = \begin{bmatrix} q_1 - q_{1e} \\ v_1 - v_{1e} \end{bmatrix} \qquad u = [\tau - \tau_e] \qquad K = [k_1 \quad k_2]$$

$$C = [1 \quad 0] \qquad y = [q_1 - q_{1e}] \qquad L = \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix}$$

HOW TO IMPLEMENT?

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

$$u = -K\hat{x}$$
$$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$$

RESET $\longrightarrow$ $\hat{x}(0)$ = some guess

$\vdots$

RUN $\longrightarrow$
$$\begin{cases} u(t) = -K\hat{x}(t) \\ \hat{x}(t+\Delta t) \approx \hat{x}(t) + \Delta t\left(A\hat{x}(t) + Bu(t) - L\left(C\hat{x}(t) - y(t)\right)\right) \end{cases}$$

$\vdots$

WHEN DOES IT WORK?

$$\dot{x} = Ax + Bu \qquad u = -K\hat{x}$$
$$y = Cx \qquad \dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$$

$$x_{err} = \hat{x} - x$$ ← does this go to zero or not?

$$\dot{x}_{err} = \dot{\hat{x}} - \dot{x}$$

$$\dot{x}_{err} = (A - LC)\,x_{err}$$

$$\dot{x} = Ax + Bu$$
$$u = -Kx$$
$$\dot{x} = (A - BK)\,x$$

$$(A - LC)^T = A^T - (LC)^T$$
$$= A^T - C^T L^T$$

HOW TO CHOOSE L ?

$$\dot{x} = (A - BK)x \qquad\qquad \dot{x}_{err} = (A - LC)\,x_{err}$$

$$0 = \det(sI - (A-BK)) \qquad\qquad 0 = \det(sI - (A-LC))$$

$$K = \text{acker}(A, B, P_{cont}) \qquad\qquad L = \text{acker}(A^T, C^T, P_{obsv})^T$$

WHEN IS OBSERVER DESIGN POSSIBLE?

$\dot{x} = (A - BK)x$

controllable when

$$\begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix}$$

is full rank

$\dot{x}_{err} = (A - LC)x_{err}$

observable when

$$\begin{bmatrix} C^T & A^TC^T & (A^T)^2C^T & \cdots & (A^T)^{n-1}C^T \end{bmatrix}$$

is full rank

$$\begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

WHAT ABOUT CONTROL ?

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

$$u = -K\hat{x}$$
$$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$$

$$x_{err} = \hat{x} - x \Rightarrow \hat{x} = x_{err} + x$$

$$\begin{aligned}\dot{x} &= Ax + Bu \\ &= Ax + B(-K\hat{x}) \\ &= Ax - BK(x_{err} + x) \\ &= (A - BK)x - BKx_{err}\end{aligned}$$

$$\dot{x}_{err} = (A - LC)x_{err}$$

$$\begin{bmatrix} x(t) \\ x_{err}(t) \end{bmatrix} =$$

F

$$\begin{bmatrix} \dot{x} \\ \dot{x}_{err} \end{bmatrix} = \begin{bmatrix} A-BK & -BK \\ 0 & A-LC \end{bmatrix} \begin{bmatrix} x \\ x_{err} \end{bmatrix}$$