Day 16

LQR

AE353

Spring 2022

Bretl

$$\dot{x} = Ax + Bu$$
$$u = -Kx$$

HOW TO FIND K ?

① Gain tuning (i.e., guess and check)

- make a small change to K
- check if all eigenvalues of A-BK have negative real part
- repeat until satisfied

② Eigenvalue placement

- choose desired eigenvalue locations
- apply "place_poles" or Ackermann's method

③ LQR (minimize a cost)

- choose weights on the cost of non-zero x and u
- choose K to minimize total, integrated cost

Linear Quadratic Regulator (LQR)

$$\underset{u_{[t_0,\infty)}}{\text{minimize}} \quad \int_{t_0}^{\infty} \left(x(t)^T Q\, x(t) + u(t)^T R\, u(t) \right) dt$$

$$\text{subject to} \quad \dot{x}(t) = A x(t) + B u(t)$$
$$x(t_0) = x_0$$

The minimizer (i.e., the input that achieves minimum cost) is

$$u(t) = -K\, x(t)$$

and the minimum (i.e., the minimum cost) is

$$x_0^T P x_0$$

where K and P can be found in python as follows:

```
def lqr(A, B, Q, R):
    P = linalg.solve_continuous_are(A, B, Q, R)
    K = linalg.inv(R) @  B.T @ P
    return K, P
```

$$\dot{x} = \overset{A}{[5]}\, x + \overset{B}{[1]}\, u$$

$$u = -\underset{K}{[k]}\, x$$

$$A - BK = [5 - k]$$

$$\underset{u_{[t_0, \infty)}}{\text{minimize}} \int_{t_0}^{\infty} \left(\overset{\text{weights}}{q}\, x(t)^2 + \overset{\text{weights}}{r}\, u(t)^2 \right) dt$$

$$\text{s.t.} \quad \dot{x}(t) = A x(t) + B u(t)$$

$$x(t_0) = x_0$$

LQR problem (for a scalar system)