

Day 14

Controllability

AE 353

Spring 2022

Bretl

ACKERMANN'S METHOD

- Compute the characteristic equation that we want:

$$(s - p_1) \cdots (s - p_n) = s^n + r_1 s^{n-1} + \cdots + r_{n-1} s + r_n$$

- Compute the characteristic equation that we have:

$$\det(sI - A) = s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n$$

- Compute the controllability matrix of the original system (and check that $\det(W) \neq 0$):

$$W = [B \quad AB \quad \cdots \quad A^{n-1}B] \quad \leftarrow$$

- Compute the controllability matrix of the transformed system:

$$W_{ccf} = [B_{ccf} \quad A_{ccf} B_{ccf} \quad \cdots \quad A_{ccf}^{n-1} B_{ccf}]$$

where

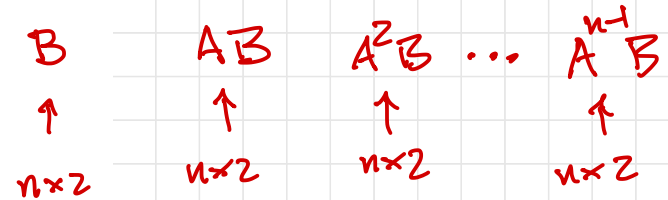
$$A_{ccf} = \begin{bmatrix} -a_1 & -a_2 & \cdots & -a_{n-1} & -a_n \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \quad B_{ccf} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

- Compute the gains for the transformed system:

$$K_{ccf} = [r_1 - a_1 \quad \cdots \quad r_n - a_n]$$

- Compute the gains for the original system:

$$K = K_{ccf} W_{ccf}^{-1}$$



$n \times 2n$

CONTROLLABLE

if 1 input: $\det(W) \neq 0$

otherwise: $\text{rank}(W) = n$

$$W = [B \quad AB \quad A^2B \quad \dots \quad A^{n-1}B]$$

of states

$$n=1 \quad W = [B]$$

$$n=2 \quad W = [B \quad AB]$$

$$n=3 \quad W = [B \quad AB \quad A^2B]$$

np-poly(p)

The system

$$\dot{x} = Ax + Bu$$

is **controllable** if and only if

$$W = [B \quad AB \quad A^2B \quad \dots \quad A^{n-1}B]$$

has full rank.