Day 14

Controllability

AE353

Spring 2022

Bretl

ACKERMANN'S METHOD

- Compute the characteristic equation that we want:

$$(s-p_1)\cdots(s-p_n) = s^n + r_1 s^{n-1} + \cdots + r_{n-1}s + r_n$$

- Compute the characteristic equation that we have:

$$\det(sI - A) = s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n$$

- Compute the controllability matrix of the original system (and check that $\det(W) \neq 0$):

$$W = \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix}$$

$$\underbrace{\underset{n\times 2}{B} \quad \underset{n\times 2}{AB} \quad \underset{n\times 2}{A^2B} \quad \cdots \quad \underset{n\times 2}{A^{n-1}B}}_{n \times 2n}$$

CONTROLLABLE

if 1 input: $\det(W) \neq 0$

otherwise: $\text{rank}(W) = n$

- Compute the controllability matrix of the transformed system:

$$W_{\text{ccf}} = \begin{bmatrix} B_{\text{ccf}} & A_{\text{ccf}}B_{\text{ccf}} & \cdots & A_{\text{ccf}}^{n-1}B_{\text{ccf}} \end{bmatrix}$$

where

$$A_{\text{ccf}} = \begin{bmatrix} -a_1 & -a_2 & \cdots & -a_{n-1} & -a_n \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \qquad B_{\text{ccf}} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

- Compute the gains for the transformed system:

$$K_{\text{ccf}} = \begin{bmatrix} r_1 - a_1 & \cdots & r_n - a_n \end{bmatrix}$$

- Compute the gains for the original system:

$$K = K_{\text{ccf}}W_{\text{ccf}}W^{-1}$$

$$W = [B \quad AB \quad A^2B \quad \cdots \quad A^{n-1}B]$$

# of states

$n = 1 \qquad W = [B]$

$n = 2 \qquad W = [B \quad AB]$

$n = 3 \qquad W = [B \quad AB \quad A^2B]$

np.poly(p)

The system

$$\dot{x} = Ax + Bu$$

is controllable if and only if

$$W = [B \quad AB \quad A^2B \quad \cdots \quad A^{n-1}B]$$

has full rank.