Day 12

Eigenvalue placement

AE353

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Bretl

The eigenvalues of a matrix are the roots of its characteristic polynomial

$$\left.\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ u &= -\begin{bmatrix} 20 & 9 \end{bmatrix} x \end{aligned}\right\}$$

$$A - BK = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 20 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix}$$

$$\det(sI - (A - BK))$$

$$= \det\left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix} \right)$$

$$= \det\left(\begin{bmatrix} s & -1 \\ 20 & s+9 \end{bmatrix} \right)$$

$$= s^2 + 9s + 20 = (s+4)(s+5) \leftarrow \quad s_1 = -4 \qquad s_2 = -5$$

One way to place eigenvalues is to equate coefficients of the characteristic polynomial

$$K = \begin{bmatrix} 6 & 5 \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$u = -\begin{bmatrix} k_1 & k_2 \end{bmatrix} x$$

← find k_1 and k_2 to put closed-loop eigenvalues at $\boxed{-2}$ and $\boxed{-3}$

What do we want?

$$(s-(-2))(s-(-3)) = (s+2)(s+3) = \boxed{s^2 + 5s + 6}$$

$k_2 = 5$

$k_1 = 6$

What do we have?

$$A - BK = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix}$$

$$\det(sI - (A-BK)) = \det\left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix} \right)$$

$$= \det\left(\begin{bmatrix} s & -1 \\ k_1 & s+k_2 \end{bmatrix} \right) = \boxed{s^2 + k_2 s + k_1}$$

$$(s - (-5 + 3j))(s - (-5 - 3j))$$

It is possible to automate the process of eigenvalue placement

See python demo