Day 7

AE353

Spring 2022

Bretl

DYNAMICS: $\dot{x} = Ax + Bu$ CONTROLLER: $u = -Kx$

CONTROL OF WHEEL ONLY

$x = \begin{bmatrix} v_2 - 2\pi \end{bmatrix}$ $u = \begin{bmatrix} \tau - 0 \end{bmatrix}$ $A = \begin{bmatrix} 0 \end{bmatrix}$ $B = \begin{bmatrix} -33 \end{bmatrix}$ $K = \begin{bmatrix} -5/33 \end{bmatrix}$

CONTROL OF PLATFORM ONLY

$x = \begin{bmatrix} q_1 - (\pi/6) \\ v_1 - 0 \end{bmatrix}$ $u = \begin{bmatrix} \tau - 0 \end{bmatrix}$ $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $K = \begin{bmatrix} 10 & 7 \end{bmatrix}$

CONTROL OF PLATFORM AND WHEEL (WITH GRAVITY)

$x = \begin{bmatrix} q_1 - \pi \\ v_1 - 0 \\ v_2 - 0 \end{bmatrix}$ $u = \begin{bmatrix} \tau - 0 \end{bmatrix}$ $A = \begin{bmatrix} 0 & 1 & 0 \\ 2.4525 & 0 & 0 \\ -2.4525 & 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \\ -33 \end{bmatrix}$ $K = \begin{bmatrix} 100 & 50 & 1 \end{bmatrix}$

WHAT IS "GOOD" ?

ASYMPTOTIC STABILITY

CONTROL OF WHEEL ONLY

$x = [v_2 - 2\pi]$ $\quad u = [\tau - 0]$

$v_2(t) \to 2\pi$ as $t \to \infty$

$x(t) \to 0$ as $t \to \infty$

CONTROL OF PLATFORM ONLY

$x = \begin{bmatrix} q_1 - (\pi/6) \\ v_1 - 0 \end{bmatrix}$ $\quad u = [\tau - 0]$

$q_1(t) \to \pi/6$, $v_1(t) \to 0$ as $t \to \infty$

$x(t) \to 0$ as $t \to \infty$

CONTROL OF PLATFORM AND WHEEL (WITH GRAVITY)

$x = \begin{bmatrix} q_1 - \pi \\ v_1 - 0 \\ v_2 - 0 \end{bmatrix}$ $\quad u = [\tau - 0]$

$q_1(t) \to \pi$, $v_1(t) \to 0$, $v_2(t) \to 0$ as $t \to \infty$

$x(t) \to 0$ as $t \to \infty$

WHICH CHOICES OF K ARE "GOOD"?

"GOOD" = make the closed-loop system stable

CONTROL OF WHEEL ONLY

$K = [-5 \; 33]$ $K = [-1 \; 33]$ ~~$K = [1 \; 33]$~~

CONTROL OF PLATFORM ONLY

$K = [10 \quad 7]$ ~~$K = [-6 \quad 1]$~~ $K = [5 \quad 2]$

CONTROL OF PLATFORM AND WHEEL (WITH GRAVITY)

$K = [100 \quad 50 \quad 1]$ ~~$K = [1 \quad 1 \quad 1]$~~ ~~$K = [100 \quad 50 \quad -10]$~~

CAN WE PREDICT WHAT WILL HAPPEN WITHOUT SIMULATION?

$$\left.\begin{aligned}\dot{x} &= Ax + Bu \\ u &= -Kx\end{aligned}\right\} \Rightarrow \dot{x} = Ax + B(-Kx)$$

$$\boxed{\dot{x} = (A - BK)x}$$

$\dot{x} = (a - bk)x \qquad x(t) = e^{(a-bk)t} x(0)$

SCALAR EXPONENTIAL

$$e^m = 1 + m + \frac{1}{2}m^2 + \frac{1}{6}m^3 + \cdots = \sum_{n=0}^{\infty} \frac{1}{n!} m^n$$

$\dot{x} = (A - BK)x \qquad x(t) = e^{(A-BK)t} x(0)$

MATRIX EXPONENTIAL

$$e^M = I + M + \frac{1}{2}M^2 + \frac{1}{6}M^3 + \cdots = \sum_{n=0}^{\infty} \frac{1}{n!} M^n$$

CAN WE PREDICT WHAT WILL HAPPEN WITHOUT FINDING $x(t)$?

The closed-loop system

$$\dot{x} = (A - BK)x$$

is asymptotically stable if and only if all eigenvalues of

$$A - BK$$

have negative real part.