Day 3

Linearization

AE353

Spring 2022

Bretl

While we are waiting to start...

1. Go to the schedule page on the course website: https://tbretl.github.io/ae353-sp22/schedule

2. Click on the “WS3” link in the activity column

3. Click on the workspace question

4. Click on “Open workspace”

Last Time

$J\ddot{q} = \tau$ → $\dot{x} = Ax + Bu$ } state-space model

$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1/J \end{bmatrix}$

$\begin{bmatrix} q - q_{des} \\ v - 0 \end{bmatrix}$ state $\quad \begin{bmatrix} \tau - 0 \end{bmatrix}$ input

equation of motion

linear state feedback

$u = -Kx$

$\tau - 0 = -1(q - q_{des}) - 1(v - 0)$

$u = -\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} q - q_{des} \\ v - 0 \end{bmatrix}$

$$(J + ml^2)\ddot{q} = \tau - mgl\cos q$$

① Rewrite as first-order ODEs

$$\dot{q} = v$$

$$\dot{v} = (J + ml^2)^{-1}(\tau - mgl\cos q)$$

② Choose equilibrium point

$$0 = v_{eq}$$

$$0 = (J + ml^2)^{-1}(\tau_{eq} - mgl\cos q_{eq}) \leftarrow$$

$q_{eq} = q_{des}$

$\tau_{eq} = mgl\cos q_{des}$

③ Define state and input

$$x = \begin{bmatrix} q - q_{des} \\ v - 0 \end{bmatrix} \qquad u = \begin{bmatrix} \tau - mgl\cos q_{des} \end{bmatrix}$$

$$\begin{bmatrix} \dot{q} \\ \dot{v} \end{bmatrix} = \underbrace{\begin{bmatrix} v \\ (J+ml^2)^{-1}(\tau - mgl\cos q) \end{bmatrix}}_{f(q, v, \tau)}$$

$$\dot{x} = \frac{d}{dt}\begin{bmatrix} q - q_{des} \\ v - 0 \end{bmatrix} = \begin{bmatrix} \dot{q} \\ \dot{v} \end{bmatrix} = f(q, v, \tau)$$

$$\approx f(q_{eq}, v_{eq}, \tau_{eq}) + \frac{\partial f}{\partial q}\bigg|_{(q_{eq}, v_{eq}, \tau_{eq})}(q - q_{eq}) + \frac{\partial f}{\partial v}\bigg|_{(q_{eq}, v_{eq}, \tau_{eq})}(v - v_{eq}) + \frac{\partial f}{\partial \tau}\bigg|_{(q_{eq}, v_{eq}, \tau_{eq})}(\tau - \tau_{eq})$$

$mgl\cos q_{des}$

$$= \begin{bmatrix} 0 \\ (J+ml^2)^{-1}mgl\sin q_{des} \end{bmatrix}(q - q_{des}) + \begin{bmatrix} 1 \\ 0 \end{bmatrix}(v - 0) + \begin{bmatrix} 0 \\ (J+ml^2)^{-1} \end{bmatrix}(\tau -)$$

$$= \underbrace{\begin{bmatrix} 0 & 1 \\ (J+ml^2)^{-1}mgl\sin q_{des} & 0 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} q - q_{des} \\ v - 0 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 \\ (J+ml^2)^{-1} \end{bmatrix}}_{B}\underbrace{\begin{bmatrix} \tau - mgl\cos q_{des} \end{bmatrix}}_{u}$$