Day 3

Linearization

AE 353
Spring 2022
Bret1
While we are waiting to start...

1. Go to the schedule page on the course website: 
   https://tbretl.github.io/ae353-sp22/schedule

2. Click on the “WS3” link in the activity column

3. Click on the workspace question

4. Click on “Open workspace”
LAST TIME

\[ \dot{q}_b = \tau \]

\[ \dot{x} = Ax + Bu \]

\[ r - o = -1(q - q_{des}) - 1(v - o) \]

\[ u = -Kx \]
(J + ml^2) \ddot{q} = \tau - mgl \cos q

1. Rewrite as first-order ODEs

\dot{q} = \nu

\dot{\nu} = (J + ml^2)^{-1} (\tau - mgl \cos q)

2. Choose equilibrium point

\begin{align*}
O &= \nu_{eq} \\
O &= (J + ml^2)^{-1} (\nu_{eq} - mgl \cos q_{eq})
\end{align*}

\begin{align*}
q_{eq} &= q_{des} \\
\nu_{eq} &= mgl \cos q_{des}
\end{align*}

3. Define state and input

\begin{align*}
x &= \begin{bmatrix} q - q_{des} \\ \nu - O \end{bmatrix} \\
u &= \begin{bmatrix} \tau - mgl \cos q_{des} \end{bmatrix}
\end{align*}
\[
\begin{bmatrix}
\dot{q} \\
\dot{v}
\end{bmatrix} = \begin{bmatrix} v \\
(\mathcal{J}+ml^2)^{-1}(q - mgl\cos q_0)
\end{bmatrix}
\]

\[f(q, v, \tau)\]

\[x = \frac{d}{dt} \begin{bmatrix} q - g\text{des} \\
v - 0
\end{bmatrix} = \begin{bmatrix} \dot{q} \\
\dot{v}
\end{bmatrix} = f(q, v, \tau)\]

\[= \begin{bmatrix} 0 \\
(\mathcal{J}+ml^2)^{-1}mgl\sin q_{\text{des}}
\end{bmatrix}(q - g\text{des}) + \begin{bmatrix} 0 \\
1
\end{bmatrix}(v - 0) + \begin{bmatrix} 0 \\
(\mathcal{J}+ml^2)^{-1}
\end{bmatrix}(q - mgl\cos q_0)\]