Day 2

Velocity control example + State space models

AE353

Spring 2022

Bretl

While we are waiting to start...

1. Go to the schedule page on the course website: https://tbretl.github.io/ae353-sp22/schedule

2. Click on the “WS2” link in the activity column

3. Click on the “Workspace: Wheel demo” question

4. Click on “Open workspace”

moment of inertia

$J\ddot{q} = \tau$ → $\dot{x} = Ax + Bu$ ← "state space model"

wheel torque

$\frac{d^2}{dt^2}$(wheel angle)

state: x input: u

① Rewrite EOMs as a set of first-order ODEs

$$\dot{q} = v$$
$$\dot{v} = (1/J)\tau \quad \rightarrow \quad \begin{bmatrix} \dot{q} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \\ (1/J)\tau \end{bmatrix}$$

② Choose an equilibrium point

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} v_{eq} \\ (1/J)\tau_{eq} \end{bmatrix} \quad \begin{matrix} \leftarrow v_{eq} = 0 \\ \leftarrow \tau_{eq} = 0 \end{matrix} \Big\} \; q_{eq} = q_{des}$$

③ Define state and input

$$x = \begin{bmatrix} q - q_{des} \\ v - 0 \end{bmatrix} \qquad u = [\tau - 0]$$

④ Rewrite ODEs in state-space form

A x B u

$$\dot{x} = \begin{bmatrix} \dot{q} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \\ (1/J)\tau \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ (1/J) \end{bmatrix} u$$

$$x = \begin{bmatrix} q - q_{des} \\ v \end{bmatrix}$$ (2×1)

$$u = [T]$$ (1×1)

$$\dot{x} = \begin{bmatrix} \dot{q} \\ \dot{v} \end{bmatrix} = Ax + Bu$$

$\dot{x}$: 2×1; A: 2×2

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} q - q_{des} \\ v \end{bmatrix}$$

$$\begin{bmatrix} 0(q - q_{des}) + 1(v) \\ 0(q - q_{des}) + 0(v) \end{bmatrix}$$