

AE353 (Spring 2021)

Day 31

T. Bretl

Optimal
Observer
Design

OPTIMALITY?

Controller

minimize
 $u_{[t_0, \infty)}$

subject to

$$\int_{t_0}^{\infty} (x(t)^T Q_c x(t) + u(t)^T R_c u(t)) dt$$

$$\begin{aligned} \dot{x}(t) &= A x(t) + B u(t) & \text{for all } t \in [t_0, \infty) \\ x(t_0) &= x_0 \end{aligned}$$

$P = \text{solve_continuous_are}(A, B, Q_c, R_c)$
 $K = R_c^{-1} B^T P$

LQR

Observer

minimize
 \hat{x}, d, n

subject to

$$\int_{-\infty}^{t_1} (n(t)^T Q_o n(t) + d(t)^T R_o d(t)) dt$$

$$\begin{aligned} \dot{x}(t) &= A x(t) + B u(t) + d(t) & \text{for all } t \in (-\infty, t_1] \\ y(t) &= C x(t) + n(t) \end{aligned}$$

$P = \text{solve_continuous_are}(A^T, C^T, R_o^{-1}, Q_o^{-1})$
 $L = P C^T Q_o$

$$\dot{\hat{x}} = (A\hat{x} + Bu) - L(C\hat{x} - y)$$

dynamic
model

$$\dot{x} = Ax + Bu$$

sensor
model

$$y = Cx$$

$$\dot{\hat{x}} - (A\hat{x} + Bu) = d$$

$$y - C\hat{x} = n$$