AE353 (Spring 2021)

Day 30

T. Bretl

How to linearize a nonlinear sensor model

WHAT IF MEASUREMENT IS NONLINEAR?

$$\begin{bmatrix} \dot{q} \\ \dot{v} \end{bmatrix} = f(q, v, \tau) \longrightarrow \dot{x} = Ax + Bu$$

$$x = \begin{bmatrix} q - q_e \\ v - v_e \end{bmatrix} \quad u = [\tau - \tau_e]$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$z = g(q, v, \tau)$$

$$\approx g(q_e, v_e, \tau_e) + \left.\frac{\partial g}{\partial q}\right|_{(q_e, v_e, \tau_e)} (q - q_e) + \left.\frac{\partial g}{\partial v}\right|_{(\cdot)} (v - v_e) + \left.\frac{\partial g}{\partial \tau}\right|_{(\cdot)} (\tau - \tau_e)$$

$$\underbrace{z - g(q_e, v_e, \tau_e)}_{y} \approx \underbrace{\begin{bmatrix} \left.\frac{\partial g}{\partial q}\right|_{(q_e, v_e, \tau_e)} & \left.\frac{\partial g}{\partial v}\right|_{(q_e, v_e, \tau_e)} \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} q - q_e \\ v - v_e \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} \left.\frac{\partial g}{\partial \tau}\right|_{(q_e, v_e, \tau_e)} \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} \tau - \tau_e \end{bmatrix}}_{u}$$

$\xi_e = 0, \quad v_e = 0$

$z = \cos q_6$

$y = [z - \cos q_{6e}] = [z - 1]$

$C = \begin{bmatrix} \dfrac{\partial \cos q_6}{\partial q_6} & \dfrac{\partial \cos q_6}{\partial v} \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \end{bmatrix}$

$W_o = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

not full rank
so
not observable!

$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$z = \sin q_6$

$y = [z - \sin q_{6e}]$

$= [z - 0]$

$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

$\dfrac{\partial \sin q_6}{\partial q_6}$

$W_o = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

full rank
so
observable!