

AE353 (Spring 2021)

Day 29

Observer design  
+

T. Bretl

analysis

(part 2)

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned}$$

$$u = -K\hat{x}$$

$$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$$

← controller

← observer

implement as

$$\left\{ \begin{array}{l} \hat{x}(0) = \text{some guess} \\ \vdots \\ \hat{x}(t+\Delta t) = \hat{x}(t) + \Delta t (A\hat{x}(t) + Bu(t) - L(C\hat{x}(t) - y(t))) \end{array} \right.$$

# WHEN DOES IT WORK?

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned}$$

$$u = -K\hat{x}$$

← controller

$$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$$

← observer

$$x_{\text{err}} = \hat{x} - x \quad \leftarrow \text{does this go to zero or not?}$$

$$\dot{x}_{\text{err}} = \dot{\hat{x}} - \dot{x}$$

$$= A\hat{x} + \cancel{Bu} - L(C\hat{x} - y) - (Ax + \cancel{Bu})$$

$$= A(\hat{x} - x) - L(C(\hat{x} - x))$$

$\uparrow$   $Cx$   
 $\uparrow$   $x_{\text{err}}$

$$= Ax_{\text{err}} - LCx_{\text{err}}$$

$$\dot{x}_{\text{err}} = (A - LC)x_{\text{err}}$$

$\uparrow$

$$\dot{x} = (A - BK)x$$

# How to choose L?

$$\dot{x} = (A - BK)x$$

$$0 = \det(sI - (A - BK))$$

$$K = \text{acker}(A, B, p)$$

$$\dot{x}_{\text{err}} = (A - LC)x_{\text{err}}$$

$$0 = \det(sI - (A - LC))$$

$$= \det((sI - (A - LC))^T)$$

$$= \det((sI)^T - (A - LC)^T)$$

$$= \det(sI - (A^T - (LC)^T))$$

$$0 = \det(sI - (A^T - C^T L^T))$$

# WHEN IS OBSERVER DESIGN POSSIBLE?

$$\dot{x} = (A - BK)x$$

controllable when

$$\begin{bmatrix} B & AB & \dots & A^{n-1}B \end{bmatrix}$$

is full rank

$$\dot{x}_{err} = (A - LC)x_{err}$$

observable when

$$\begin{bmatrix} C^T & A^T C^T & \dots & A^{T^{n-1}} C^T \end{bmatrix}^T$$

is full rank

$$\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} = \underbrace{\begin{bmatrix} C^T & (CA)^T & \dots & (CA^{n-1})^T \end{bmatrix}^T}_{\text{is full rank}}$$

# WHAT ABOUT CONTROL?

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned}$$

$$u = -K\hat{x}$$

← controller

$$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$$

← observer

$$\dot{x}_{err} = (A - LC)x_{err} \quad \text{where} \quad x_{err} = \hat{x} - x$$

$$\dot{x} =$$

← write in terms of  
x and x<sub>err</sub>

$$\begin{bmatrix} \dot{x} \\ \dot{x}_{err} \end{bmatrix} =$$

$$\begin{bmatrix} \phantom{\dot{x}} \\ \phantom{\dot{x}_{err}} \end{bmatrix} \begin{bmatrix} x \\ x_{err} \end{bmatrix}$$

# WHAT IF MEASUREMENT IS NONLINEAR?

$$Y = \cos q$$

$$Y = \sin q$$

# OPTIMALITY?

Controller

minimize  
 $u_{[t_0, \infty)}$

$$\int_{t_0}^{\infty} (x(t)^T Q_c x(t) + u(t)^T R_c u(t)) dt$$

subject to

$$\begin{aligned} \dot{x}(t) &= A x(t) + B u(t) & \text{for all } t \in [t_0, \infty) \\ x(t_0) &= x_0 \end{aligned}$$

$$\begin{aligned} P &= \text{solve\_continuous\_are}(A, B, Q_c, R_c) \\ K &= R_c^{-1} B^T P \end{aligned}$$

Observer

minimize  
 $u_{(-\infty, t_1]}$

$$\int_{-\infty}^{t_1} (n(t)^T Q_o n(t) + d(t)^T R_o d(t)) dt$$

subject to

$$\begin{aligned} \dot{x}(t) &= A x(t) + B u(t) + d(t) & \text{for all } t \in (-\infty, t_1] \\ y(t) &= C x(t) + n(t) \end{aligned}$$

$$\begin{aligned} P &= \text{solve\_continuous\_are}(A^T, C^T, R_o^{-1}, Q_o^{-1}) \\ L &= P C^T Q_o \end{aligned}$$