AE353 (Spring 2021)

Day 29

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Observer design + analysis (part 2)

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

$$u = -K\hat{x} \quad \leftarrow \text{controller}$$
$$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y) \quad \leftarrow \text{observer}$$

implement as
$$\begin{cases} \hat{x}(0) = \text{some guess} \\ \vdots \\ \hat{x}(t+\Delta t) = \hat{x}(t) + \Delta t\left(A\hat{x}(t) + Bu(t) - L\left(C\hat{x}(t) - y(t)\right)\right) \end{cases}$$

WHEN DOES IT WORK?

$$\dot{x} = Ax + Bu \qquad u = -K\hat{x} \quad \leftarrow \text{controller}$$
$$y = Cx \qquad \dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y) \quad \leftarrow \text{observer}$$

$$x_{err} = \hat{x} - x \quad \leftarrow \text{does this go to zero or not?}$$

$$\dot{x}_{err} = \dot{\hat{x}} - \dot{x}$$
$$= A\hat{x} + \cancel{Bu} - L(C\hat{x} - \underbrace{y}_{Cx}) - (Ax + \cancel{Bu})$$
$$= A(\hat{x} - x) - L(C(\underbrace{\hat{x} - x}_{x_{err}}))$$
$$= A\,x_{err} - LC\,x_{err}$$
$$\boxed{\dot{x}_{err} = (A - LC)\,x_{err}} \quad \leftarrow \quad x_{err}(t) = e^{(A-LC)(t-t_0)}\,x_{err}(t_0)$$

$$\dot{x} = (A - BK)\,x$$

HOW TO CHOOSE L ?

$$\dot{x} = (A - BK)x$$

$$\boxed{0 = \det(sI - (A - BK))}$$

$$K = \text{acker}(A, B, p)$$

$$\dot{x}_{err} = (A - LC)x_{err}$$

$$0 = \det(sI - (A - LC))$$

$$= \det\left((sI - (A - LC))^T\right)$$

$$= \det\left((sI)^T - (A - LC)^T\right)$$

$$= \det\left(sI - (A^T - (LC)^T)\right)$$

$$\boxed{0 = \det(sI - (A^T - C^T L^T))}$$

WHEN IS OBSERVER DESIGN POSSIBLE?

$\dot{x} = (A - BK)x$

controllable when

$$\begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix}$$

is full rank

$\dot{x}_{err} = (A - LC)\,x_{err}$

observable when

$$\begin{bmatrix} C^T & A^T C^T & \cdots & {A^T}^{n-1} C^T \end{bmatrix}^T$$

is full rank

$$\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} = \underbrace{\begin{bmatrix} C^T & (CA)^T & \cdots & (CA^{n-1})^T \end{bmatrix}^T}$$

WHAT ABOUT CONTROL?

$$\dot{x} = Ax + Bu \qquad u = -K\hat{x} \quad \leftarrow \text{controller}$$
$$y = Cx \qquad \dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y) \quad \leftarrow \text{observer}$$

$\rightarrow$ $\dot{x}_{err} = (A - LC)\,x_{err}$ where $\boxed{x_{err} = \hat{x} - x}$

$\rightarrow$ $\dot{x} = Ax + B(-K\hat{x})$ ← write in terms of x and x_{err}

$$= Ax - BK(x_{err} + x)$$
$$= (A - BK)x - BK\,x_{err}$$

$\rightarrow$
$\rightarrow$
$$\begin{bmatrix} \dot{x} \\ \dot{x}_{err} \end{bmatrix} = \boxed{\begin{bmatrix} A - BK & -BK \\ 0 & A - LC \end{bmatrix}} \begin{bmatrix} x \\ x_{err} \end{bmatrix}$$

WHAT IF MEASUREMENT IS NONLINEAR?

$y = \cos q$

$y = \sin q$

OPTIMALITY?

Controller

$$\underset{u[t_0,\infty)}{\text{minimize}} \quad \int_{t_0}^{\infty} \left(x(t)^T Q_c x(t) + u(t)^T R_c u(t) \right) dt$$

subject to

$$\dot{x}(t) = A x(t) + B u(t) \qquad \text{for all } t \in [t_0, \infty)$$

$$x(t_0) = x_0$$

$$P = \text{solve\_continuous\_are}(A, B, Q_c, R_c)$$

$$K = R_c^{-1} B^T P$$

Observer

$$\underset{u(-\infty,t_1]}{\text{minimize}} \quad \int_{-\infty}^{t_1} \left(n(t)^T Q_o n(t) + d(t)^T R_o d(t) \right) dt$$

subject to

$$\dot{x}(t) = A x(t) + B u(t) + d(t) \qquad \text{for all } t \in (-\infty, t_1]$$

$$y(t) = C x(t) + n(t)$$

$$P = \text{solve\_continuous\_are}(A^T, C^T, R_o^{-1}, Q_o^{-1})$$

$$L = P C^T Q_o$$