

AE353 (Spring 2021)

Day 28

Observer design
+

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analysis

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned}$$

$$u = -K\hat{x}$$

$$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$$

← controller

← observer

implement as

$$\left\{ \begin{array}{l} \hat{x}(0) = \text{some guess} \\ \vdots \\ \hat{x}(t+\Delta t) = \hat{x}(t) + \Delta t (A\hat{x}(t) + Bu(t) - L(C\hat{x}(t) - y(t))) \end{array} \right.$$

WHEN DOES IT WORK?

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned}$$

$$u = -K\hat{x}$$

$$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$$

← controller

← observer

$$x_{err} = \hat{x} - x$$

← does this go to zero or not?

How to choose L?

$$\dot{x} = (A - BK)x$$

$$0 = \det(sI - (A - BK))$$

$$\dot{x}_{err} = (A - LC)x_{err}$$

$$0 = \det(sI - (A - LC))$$

WHAT ABOUT CONTROL?

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned}$$

$$u = -K\hat{x}$$

← controller

$$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$$

← observer

$$\dot{x}_{err} = (A - LC)x_{err} \quad \text{where} \quad x_{err} = \hat{x} - x$$

$$\dot{x} =$$

← write in terms of
 x and x_{err}

$$\begin{bmatrix} \dot{x} \\ \dot{x}_{err} \end{bmatrix} =$$

$$\begin{bmatrix} \phantom{\dot{x}} \\ \phantom{\dot{x}_{err}} \end{bmatrix} \begin{bmatrix} x \\ x_{err} \end{bmatrix}$$

WHEN IS OBSERVER DESIGN POSSIBLE?

$$\dot{x} = (A - BK)x$$

controllable when

$$[B \quad AB \quad \dots \quad A^{n-1}B]$$

is full rank

$$\dot{x}_{err} = (A - LC)x_{err}$$

observable when