AE353 (Spring 2021)

Day 28

Observer design + analysis

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$$\dot{x} = Ax + Bu$$
$$y = Cx$$

$$u = -K\hat{x} \quad \leftarrow \text{controller}$$
$$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y) \quad \leftarrow \text{observer}$$

implement as
$$\left\{ \begin{array}{l} \hat{x}(0) = \text{some guess} \\ \vdots \\ \hat{x}(t+\Delta t) = \hat{x}(t) + \Delta t \left(A\hat{x}(t) + Bu(t) - L\left(C\hat{x}(t) - y(t)\right)\right) \end{array} \right.$$

WHEN DOES IT WORK?

$$\dot{x} = Ax + Bu \qquad u = -K\hat{x} \quad \leftarrow \text{controller}$$

$$y = Cx \qquad \dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y) \quad \leftarrow \text{observer}$$

$$x_{err} = \hat{x} - x \quad \leftarrow \text{does this go to zero or not?}$$

$$\dot{x}_{err} = \dot{\hat{x}} - \dot{x}$$

$$= A\hat{x} + \cancel{Bu} - L(C\hat{x} - \underbrace{y}_{Cx}) - (Ax + \cancel{Bu})$$

$$= A(\hat{x} - x) - L(C(\underbrace{\hat{x} - x}_{x_{err}}))$$

$$= A\,x_{err} - LC\,x_{err}$$

$$\boxed{\dot{x}_{err} = (A - LC)\,x_{err}}$$

$$\dot{x} = (A - BK)\,x$$

HOW TO CHOOSE L?

$$\dot{x} = (A - BK)x$$

$$\boxed{0 = \det(sI - (A - BK))}$$

$K = \text{acker}(A, B, p)$

$$\dot{x}_{err} = (A - LC)x_{err}$$

$$0 = \det(sI - (A - LC))$$

$$= \det((sI - (A - LC))^T)$$

$$= \det((sI)^T - (A - LC)^T)$$

$$= \det(sI - (A^T - (LC)^T))$$

$$\boxed{0 = \det(sI - (A^T - C^T L^T))}$$

WHEN IS OBSERVER DESIGN POSSIBLE?

$\dot{x} = (A - BK)x$

controllable when

$$\begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix}$$

is full rank

$\dot{x}_{err} = (A - LC)\, x_{err}$

observable when

$$\begin{bmatrix} C^T & A^T C^T & \cdots & {A^T}^{n-1} C^T \end{bmatrix}^T$$

is full rank

$$\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} = \underbrace{\begin{bmatrix} C^T & (CA)^T & \cdots & (CA^{n-1})^T \end{bmatrix}^T}$$

WHAT ABOUT CONTROL?

$\dot{x} = Ax + Bu$
$y = Cx$

$u = -K\hat{x}$ ← controller
$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$ ← observer

$\dot{x}_{err} = (A - LC)\, x_{err}$ where $x_{err} = \hat{x} - x$

$\dot{x} =$ ← write in terms of x and x_{err}

$$\begin{bmatrix} \dot{x} \\ \dot{x}_{err} \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \begin{bmatrix} x \\ x_{err} \end{bmatrix}$$