AE353 (Spring 2021)

Day 17 "Ackermann's Method"

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TODAY:

- The big picture
- Controllable canonical form

Controllable Canonical Form (CCF)

$$A = \begin{bmatrix} [-a_1 & \cdots & -a_n] \\ [\; I_{(n-1)\times(n-1)} \;] & & [\; 0_{(n-1)\times 1} \;] \end{bmatrix} \qquad B = \begin{bmatrix} [\;1\;] \\ [\; 0_{(n-1)\times 1} \;] \end{bmatrix}$$

$$A = \begin{bmatrix} -3 & 5 \\ 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Facts

$$\det(sI - A) = s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n$$

$$\cancel{A = \begin{bmatrix} -3 & 5 \\ 2 & 1 \end{bmatrix}} \qquad \cancel{B = \begin{bmatrix} 0 \\ 3 \end{bmatrix}}$$

$$A - BK = \begin{bmatrix} [-a_1 - k_1 & \cdots & -a_n - k_n] \\ [\quad I \quad] & & [\;0\;] \end{bmatrix}$$

$$K = [k_1 \;\; k_2 \;\; \cdots \;\; k_n]$$

$$\det(sI - (A - BK)) = s^n + (a_1 + k_1)s^{n-1} + \cdots + (a_{n-1} + k_{n-1})s + (a_n + k_n)$$

Consequence

if you want $\quad s^n + r_1 s^{n-1} + \cdots + r_{n-1}s + r_n$

no symbolic computation!

then $\quad k_1 = r_1 - a_1 \quad \cdots \quad k_n = r_n - a_n \quad \leftarrow K = r - a$