

AE353 (Spring 2021)

Day 15

Eigenvalue
Placement

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$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad K = [k_1 \quad k_2]$$

$$\left. \begin{array}{l} \dot{x} = Ax + Bu \\ u = -Kx \end{array} \right\} \dot{x} = (A - BK)x$$

$$\begin{aligned} A - BK &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} [k_1 \quad k_2] \\ &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix} \end{aligned}$$

$$A - BK = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix}$$

$$\det((A - BK) - sI) = 0$$

$$\det\left(\begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix} - \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}\right) = \det\left(\begin{bmatrix} -s & 1 \\ -k_1 & -k_2 - s \end{bmatrix}\right)$$

$$= s(k_2 + s) + k_1 = s^2 + k_2 s + k_1$$

$$s_1 = -2$$

$$s_2 = -3$$

$$0 = 4 - 2k_2 + k_1$$

$$0 = 9 - 3k_2 + k_1$$

$$0 = 5 - k_2$$

$$\Rightarrow k_2 = 5$$

$$k_1 = 6$$

$$(s + 2)(s + 3) = s^2 + 5s + 6$$